# Rocking of Foundations on Improved Soil: Application to 1-dof and Frame Structures

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ABSTRACT: Rocking isolation has recently been proposed as an alternative foundation design scheme, in which inelastic footing response is used as a means of protecting the superstructure. Such a response, may be desirable as it bounds the inertia forces transmitted onto the superstructure. Yet it incorporates the peril of unacceptable settlements in case of a low static factor of safety  $FS_V$ . Therefore an adequately large  $FS_V$  must be achieved in order to ensure that rocking is materialized through uplifting rather than footing settlement. Given that soil properties are seldom well known in engineering practice, guaranteeing a target  $FS_V$  value may constitute a tedious task. Therefore, this paper investigates the use of "shallow soil improvement" as an alternative approach in order to release the design from the jeopardy caused by an unforeseen inadequate  $FS_V$ . The paper studies the response of a simple 1-dof oscillator and a rocking-isolated 1-bay 2-storey frame on two-layered soil profile consisting of a stiff surface layer overlying a weak homogeneous soil stratum. Analyses were conducted employing the finite element method and involved monotonic and cyclic push-over tests and dynamic time-history analyses. It is shown that the existence of even a shallow surface layer enhances the seismic performance of the system by reducing its residual settlements.

## 1 INTRODUCTION

The idea of "rocking isolation" [Mergos & Kawashima, 2005] has recently been proposed as an alternative seismic design philosophy in which soil failure is used as a "fuse" for the superstructure. According to this design scheme, in contrast to conventional capacity design the foundation is deliberately "under-dimensioned" to promote rocking, thus limiting the inertia forces transmitted onto the superstructure. The potential effectiveness of such a design scheme has been explored analytically by, among others, [Yim & Chopra, 1984; Martin & Lam, 2000; Pecker & Pender, 2000; Faccioli et al., 2001; Harden and Hutchinson, 2006; Kawashima et al., 2007; Apostolou et al., 2007; Paolucci et al., 2008; Chatzigogos et al., 2009; Anastasopoulos et al., 2010] and experimentally [Kutter et al., 2003; Gajan and Kutter, 2008; Anastasopoulos, 2011a] for an idealized RC bridge pier, and for idealized 2storey RC frame structures [Gelagoti et al., 2011a; 2011b]. These studies have shown that the "reversal" of capacity design may substantially increase the safety margins against collapse, although it may incur increased settlement or residual foundation rotation.

In order to avoid such residual deformations, it is essential that the safety factor against static (vertical) loads  $FS_V$  be maintained adequately large. As schematically illustrated in **Figure 1**, when the  $FS_V$  is relatively large, the foundation responds to strong seismic shaking mainly through uplifting, involving only minimal soil yielding underneath the footing. It is therefore obvious that in order to promote uplifting against a sinking dominated footing response, a rather high safety factor must be achieved. In turn, ensuring an adequately high  $FS_V$  presupposes that soil properties are accurately known during the design stage: a rather overoptimistic assumption in engineering practice, which could possibly hinder the generalized applicability of rocking isolation limiting it only to cases where an extensive soil investigation is (or may become) available. However, this obstacle may possibly be overcome due to the very nature of the foundation rocking mechanism (Gazetas and Kavvadas, 1994; Poulos,2001) which only mobilizes a shallow stress bulb within the soil (**Fig.2**). In view of this, this paper aims to lift the limitations in the application of rocking isolation by investigating the potential effectiveness of shallow soil improvement, a concept commonly applicable in geotechnical engineering as a means to increase soil strength and reduce settlements. The efficacy of shallow soil improvement is first explored on a rocking 1-dof system, and subsequently extended to more complex 1-bay 2-storey frame structures through a series of non-linear numerical analyses as described in the ensuing.



**Figure 1**. Schematic illustration of the rocking response of a surface foundation subjected to combined (M, Q, N) loading for large and low values of  $FS_{\nu}$ .



**Figure 2.** Illustration of the shallow nature of the rocking mechanism: contours of plastic strain of a lightly-loaded ( $FS_v = 5.31$ ) footing founded on a soft profile (Su = 50 kPa) mitigated with an shallow crust (z/B = 0.5) of higher strength (Su = 150 kPa).

## 2 ANALYSIS METHODOLOGY AND NUMERICAL MODELLING

The finite element method (*Abaqus FE code*) is employed in the ensuing in order to study the rocking behavior of 1-dof systems. **Figure 3a** depicts the finite element model used. The soil–structure system is studied under plane-strain conditions taking account of both material (*soil*) and geometric (*uplifting* and *P*- $\delta$  effects) nonlinearities. The soil and the footing are modeled with quadrilateral continuum elements. An elastic beam element is used for the superstructure and a mass element is located at height *h* above the footing base. The foundation is connected to the soil with special interface elements permitting detachment from the supporting ground. A considerably large coefficient of friction ( $\mu = 0.7$ ) at the soil–footing interface has been adopted.

Soil behavior is modeled through a nonlinear kinematic hardening model, with Von Mises failure criterion and associated flow rule [Anastasopoulos et. al., 2011b]. The constitutive model has been va-

lidated against experimental data and is appropriate for clay under undrained conditions which is a reasonable simplification for the problem studied herein. The evolution law of the model consists of two components: a nonlinear kinematic hardening component, which describes the translation of the yield surface in the stress space (defined through the "backstress" parameter  $\alpha$ ), and an isotropic hardening component, which defines the size of the yield surface  $\sigma_0$  as a function of plastic deformation.



**Figure 3**. Effect of soil properties on the rocking response of a rigid 1-dof oscillator : (a) Problem description and F.E mesh. Computed foundation response : (b) Moment-rotation and (c) settlement-rotation envelope for two extreme cases; the ideally stiff profile (Su = 150 kPa) and the "actual" low-strength profile (Su = 50 kPa). Note the initial settlement due to the vertical loading has been sub-tracted.

# 3 EFFECTIVENESS OF SOIL IMPROVEMENT ON A ROCKING 1-DOF SYSTEM

#### Monotonic and Cyclic Loading

This section aims at identifying the main mechanisms associated with foundation rocking on improved soil profile. To this end, a rather extreme example problem is considered hereafter in order to examine the adequacy of shallow soil improvement. A low  $FS_v$  system ( $FS_v \approx 2.6$ ) is compared to a high FSv one ( $FS_v \approx 5.6$ ) in order to highlight the difference in their response to combined moment and shear force loading. The former is represented by a rigid 1-dof oscillator of H/b = 3 founded on a footing of B = 1.4 m on soil of undrained shear strength  $S_u = 50$  kPa. The latter is accomplished when the same oscillator lies on an Su =150 kPa clay stratum.

**Figures 3b and c**, plot FE analyses results of the response to monotonic lateral push-over loading of the two systems on homogeneous clay layers. The differences are not as striking in terms of Moment-rotation response (**Fig. 3b**): although the undrained shear strength drops from 150 to only 50 kPa, both the toppling angle of the oscillator and its moment capacity are only marginally affected. Yet, the picture is substantially modified in terms of vertical displacement vs rotation plots: when the high FS<sub>V</sub> foundation is subjected to lateral loading, it uplifts almost instantly while, on the other hand, when FS<sub>V</sub> is low, the foundation tends to sink within the low-strength soil even for rotation amplitudes as high as 0.015 rad (**Fig. 3c**).

Evidently, in case the actual factor of safety is lower than  $FS_V \approx 5.6$ , this particular system will experience augmented settlements which, in the extreme case examined (S<sub>u</sub>=50 instead of 150) would definitely jeopardize the design. Based on the reasoning of the previous sections, this peril could be avoided by means of soil improvement in a shallow depth beneath the footing. In the example examined here, the improved profile is assumed to be consisting of a shallow layer of thickness d= 0.5B and S<sub>u</sub> = 150kPa, overlying a low strength material of S<sub>u</sub> = 50kPa (**Figure 3a**).

Encouragingly, the oscillator on the improved layer now manifests a behavior practically equivalent to that achieved on the  $S_u = 150$  kPa profile, both in terms of moment-rotation (**Fig. 4a**) and settlement rotation curves (**Fig. 4b**). Notice, that this superior behavior of the oscillator on the improved profile is not a product of an increase of the global safety factor against vertical loads: this could hardly be accomplished by means of such a shallow improvement; yet it is the product of harshly limiting soil yielding underneath the footing (a phenomenon of only shallow effect as discussed earlier).



**Figure 4.** Effectiveness of shallow soil improvement on the rocking response of 1-dof oscillator: (a) Moment (M) - rotation ( $\theta$ ) and (b) settlement (w) – rotation ( $\theta$ ) envelopes for the three different soil profiles and (c) a zoomed view of the w- $\theta$  response for  $|\theta| < 0.015$  rad. Note that the initial settlement due to the vertical loading had been subtracted.

Notwithstanding the effectiveness of shallow soil improvement in macroscopically ensuring a behavior equivalent to that of a higher  $FS_v$  system, it is essential to mention that its actual effectiveness is in fact also dependent upon the magnitude of the imposed loading. **Figure 4c** plots a detailed view of the settlement rotation graphs of the three systems examined in an attempt to shed light in the behavior of the systems under low-amplitude of imposed rotation. Evidently, the  $S_u=50$  kPa system will experience settlement under rotation amplitudes up to 0.015 rad, while the shallow improvement would limit this zone to only 0.004 rad. (Note that for the actual high FS<sub>v</sub> system the threshold rotation drops to mere 0.0015 rad). This however, implies that although the oscillator will uplift once the imposed rotation exceeds 0.004 rad, it will also settle under lower rotation amplitudes. This implication is luminously reflected on **Figure 5**, which compares the behavior of the oscillator under imposed rotation cycles of amplitude 0.004 rad and 0.04 rad. In the former case, the settlement reaches 5% of the imposed displacement while this ratio is diminished to a mere 0.5% for the latter one.

It is obvious that although foundation rocking may be essential for the survival of structures under intense seismic events, this observation may question the effectiveness of such a foundation scheme under low intensity earthquakes which may contain several low-amplitude pulses. This question is investigated in the following sections which study the effect of shallow soil improvement in a much more complex rocking-isolated frame structure on shallow footings subjected to cyclic and seismic loading.



**Figure 5.** Investigation of the effectiveness of shallow soil improvement when the example 1-dof oscillator is subjected to cyclic loading of low ( $\theta = 0.004$  rad) and high amplitude ( $\theta = 0.04$  rad). Settlements are normalized with the imposed horizontal displacement  $\delta$ , while rotations with the maximum achieved rotation ( $\theta_{max}$ ).

## 4 ROCKING ISOLATION OF A FRAME STRUCTURE ON IMPROVED SOIL

The analyses presented in the ensuing refer to a specific example frame investigated by *Gelagoti et. al.*, *2011*. It refers to a fairly simplified urban residential structure founded on a stiff clay layer (of  $S_u = 150$  kPa). It consists of a 1-bay 2-storey reinforced concrete frame with a span of 5 m, ground floor height of 4 m, and first floor height of 3m. The superstructure has been designed with a conventional structural analysis software in accordance to the Seismic Eurocode EC8 [2000] and the Greek Reinforced Concrete Code (EKOS 2000), for a design acceleration  $A_d = 0.36$  g, and a behavior factor q = 3.5. The adopted dead and live loads (g = 1.3 kN/m<sup>2</sup> and q = 2 kN/m<sup>2</sup>) are typical values for residential buildings.

The authors compared the seismic performance of a conventionally designed foundation (with square footings of B = 1.7 m) to a specific rocking-isolation alternative (with smaller footings of B = 1.7 m) to a specific rocking-isolation alternative (with smaller footings of B = 1.7 m) to a specific rocking-isolation alternative (with smaller footings of B = 1.7 m) to a specific rocking-isolation alternative (with smaller footings of B = 1.7 m) to a specific rocking-isolation alternative (with smaller footings of B = 1.7 m) to a specific rocking-isolation alternative (with smaller footings of B = 1.7 m) to a specific rocking-isolation alternative (with smaller footings of B = 1.7 m) to a specific rocking-isolation alternative (with smaller footings of B = 1.7 m) to a specific rocking-isolation alternative (with smaller footings of B = 1.7 m) to a specific rocking-isolation alternative (with smaller footings of B = 1.7 m) to a specific rocking-isolation alternative (with smaller footings of B = 1.7 m) to a specific rocking-isolation alternative (with smaller footings of B = 1.7 m) to a specific rocking-isolation alternative (with smaller footings of B = 1.7 m) to a specific rocking-isolation alternative (with smaller footings of B = 1.7 m) to a specific rocking-isolation alternative (with smaller footings of B = 1.7 m) to a specific rocking-isolation alternative (with smaller footings of B = 1.7 m) to a specific rocking-isolation alternative (with smaller footings of B = 1.7 m) to a specific rocking-isolation alternative (with smaller footings of B = 1.7 m) to a specific rocking-isolation alternative (with smaller footings of B = 1.7 m) to a specific rocking-isolation alternative (with smaller footings of B = 1.7 m) to a specific rocking-isolation alternative (with smaller footing isolation alternativ

1.4 m, **Fig. 6a**). In this latter case footings were designed so that their moment capacity ( $M_{ult}$ ) is smaller than that of the corresponding column. Hence, when the earthquake demand exceeds the footing capacity of the foundation, uplift is promoted. In all cases examined the Safety Factor against vertical loads was adequately high (FS<sub>v</sub> > 5) so that foundation uplifting be prevailing and soil yielding impeded.

Through static pushover and nonlinear dynamic time-history analysis (using an ensemble of 24 strong motion records), the performance of the rocking–isolated alternative was found to be advantageous in very strong seismic shaking, well in excess of the design limits: it survives the earthquake demand sustaining non-negligible but repairable damage to its beams and non-structural elements (infill walls, etc.).

The present paper aims to investigate the effects of the possible over-prediction of the available soil strength (as identified previously) as well as the efficacy of shallow soil improvement (as a means to overcome them), for the case of the rocking-isolated alternative. Analyses have again been performed utilizing the FE code ABAQUS (**Figure 6b**). The seismic excitation (i.e acceleration time history) is applied at the base of the model. Free field conditions are applied at the two lateral boundaries of the model. The reinforced concrete constitutive model has been properly calibrated to simulate the non-linear moment–curvature response of the superstructure reinforced concrete members (see Gelagoti et. al. 2011a)



**Figure 6.** Rocking isolated frame founded on an improved 2-layered profile: (a) Geometry and member properties and (b) finite element model configuration.

#### 4.1 Response to monotonic and Cyclic Loading

Initially, the frame have been subjected to slow cyclic displacement-controlled push–over loading in the horizontal direction. Displacement is imposed on the upper left node of the frame, and consists of 10 cycles of amplitude  $\delta = 1.2$  m. This value corresponds to 75%  $\delta_u$ , where  $\delta_u$  is the toppling displacement of the particular frame.

**Figure 7** compares the response after the 1<sup>st</sup> and after the 9<sup>th</sup> cycle in terms of contours of produced plastic strains for the two examined systems: (a) homogeneous soil with  $S_u = 50$  kPa and (b) two-layered with a surface layer of thickness d / B = 0.5 and undrained shear strength  $S_{u1} = 150$  kPa. Once more, it is apparent that the existence of the improved zone drastically reduces the plastification underneath the footings (**Fig. 6a**) and limits the rate of settlement accumulation. Even after the 9<sup>th</sup> cycle of loading, plastification is restricted within the mitigation zone without penetrating the underlying weak soil stratum.

#### 4.2 Response to moderately strong seismic shaking

The aim of these analyses was to determine the response of the system under different seismic excitations and, through this procedure, estimate the adequate soil improvement depth. Initially, the frame was subjected to relatively moderate seismic excitations (i.e. within its design limits). The response of the frame on improved soil (of depth d / B = 0.5 and 1.0) is compared to its response when founded on: the unimproved homogeneous soil profile of  $S_u = 50$  kPa (FS<sub>V</sub> = 2.6) and the "target" case of a competent profile of  $S_u = 150$  kPa (FS<sub>V</sub>  $\approx 5.6$ ).

**Figure 8** compares the evolution of settlements as a function of rotation angle of the left footing for the four systems examined, when the model is subjected to the Duzce180 record (Duzce, Turkey 1999 earthquake). Indeed, the response of the  $FS_V \approx 2.6$  footing deviates substantially from the target  $FS_V \approx 5.6$  response: the footing accumulates settlement *w* during each strong motion cycle, reaching a peak value of 4.5 cm instead of a mere 0.5 cm in the high  $FS_v$  case. Such a high unanticipated settlement under the design earthquake definitely questions the serviceability of the frame and should be avoided. Quite encouragingly, it is seen (**Figure 8b**) that the use of an improved layer of depth only d/B = 0.5 significantly reduces the settlements, yet not approaching the minimal settlement developed in the target homogeneous profile (**Fig. 8d**). The desired behavior is better captured when the improved crust's depth increases to d / B = 1 (**Figure 8c**), which practically creates the necessary conditions to ensure a rather efficient uplifting response of the foundation.



**Figure 7.** Frame subjected to slow cyclic horizontal loading: comparison of the distribution of plastic deformations produced after (a) the first and (b) the ninth cycle of loading

# 4.3 Response to very strong seismic shaking

The effectiveness of shallow mitigation becomes palpably more impressive in case of the frame subjected to the Tabas (Tabas, Iran 1981) which overly exceeds the structure's design spectrum. The record is characterized by a multitude of strong motion cycles while its PGA exceeds 0.81 g. The evolution of settlements as a function of the rotation angle when the frame is founded on improved soil is illustrated in **Figures 9b and c**. In case of the weak (FS<sub>v</sub> = 2.6) profile, the under-designed footings of the frame accumulate severe differential settlement (reflected in the developed rotation) which gradually causes the frame to practically collapse. Apparently, the sequence of many strong motion cycles produces significant plastification extending to large soil depths which, in turn, brings about irrecoverable foundation (and structural) distortion. The beneficial effect of using an improved surface layer

with depth ratio just d / B = 0.5 in preventing the collapse of the building becomes obvious: it limits extent of soil yielding and aborts the development of permanent rotation which is responsible for the distortion of the superstructure (**Figure 9b**). The behavior is further improved when the improvement depth is d/B = 1. The foundation response tends to imitate that of the target (FS<sub>V</sub>  $\approx$  5.6) profile. Although the rocking–induced residual settlement of the foundation is higher than in the homogeneous S<sub>u</sub>=150 kPa profile (3cm instead of 2cm), it is considered as a relatively fair price to pay.



**Figure 8.** Frame excited by the Duzce 1999 record: comparison of vertical displacement versus rotation (w- $\theta$ ) loops for the case of (a) homogeneous  $S_u = 50 \text{ kPa}$ , (b) two layered profile d/B= 0.5, (c) two layered profile d/B= 1; and (d) homogeneous  $S_u = 150 \text{ kPa}$ .



**Figure 9.** Frame excited by the Tabas 1981 record: comparison of vertical displacement versus rotation (w- $\theta$ ) loops for the case of (a) homogeneous  $S_u = 50$  kPa, (b) two layered profile d/B= 0.5, (c) two layered profile d/B= 1; and (d) homogeneous  $S_u = 150$  kPa.

#### 4.4 Response to various recorded time-histories: Summary of Results

The dynamic response of the system has been simulated employing nonlinear dynamic time history analysis. A quite comprehensive database of 20 recorded time-histories was used as input to assess the seismic performance of the systems under different earthquake scenarios. The selected records incorporate the effect of a wide range of strong-motion parameters such as *PGA*, *PGV*, *SA*, *SV*, frequency content, number of strong motion cycles, duration.

**Figure 10** displays comparative collective results of the settlement for the left footing. Obviously, the use of a surface layer of depth only d / B = 0.5, significantly reduces the residual settlement for all seismic excitations although the target behavior of the homogeneous  $S_u = 150$  kPa case is not perfectly imitated. Further increase of the soil improvement depth to d / B = 1 further reduces the residual settlements while the foundation behavior resembles that achieved in case of the target profile.

#### **5** CONCLUSIONS

This paper has studied the response of a simple 1-dof oscillator and a rocking-isolated 1-bay 2-storey frame on two-layered soil profile consisting of a stiff surface layer overlying a weak homogeneous soil stratum. For the case of 1-dof systems, it was shown that shallow soil improvement is very effective in drastically limiting the soil deformations associated with foundation rocking on a weak soil profile al-though its effectiveness may be less pronounced in case of low-amplitude earthquakes.

For the case of the more complex 2-storey frame structure examined, it is concluded that the use of a shallow improved soil layer of depth d/B = 1 is sufficient to reduce the risk of settlement associated with uncertainties in the proper estimation of soil properties, having a favorable effect for the majority of the examined seismic records limiting settlement and damage in structural members.



Figure 10. Conclusive results. Comparison of the residual settlement for all investigated earthquake scenarios for all the examined scenarios: (a) homogeneous  $S_u = 50$  kPa, (b) two layered profile d/B = 0.5, (c) two layered profile d/B = 1; and (d) homogeneous  $S_u = 150$  kPa

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## REFERENCES

Anastasopoulos I., Gelagoti F., Kourkoulis R., G. Gazetas, (2011b) "Simplified Constitutive Model for Simulation of Cyclic Response of Shallow Foundations : Validation against Laboratory Tests", Journal of Geotechnical and Geoenvironmetal Engineering, ASCE (in print *doi:10.1061/(ASCE)GT.1943-5606.0000534*)

Anastasopoulos I., Gazetas G., Loli M., Apostolou M, Gerolymos N. (2010) "Soil failure can be used for seismic protection of structures", Bull Earthquake Eng. Vol.8, pp. 309-326 Anastasopoulos I., Kourkoulis R., Gelagoti F., Papadopoulos E. (2011a) "Metaplastic Rocking

Anastasopoulos I., Kourkoulis R., Gelagoti F., Papadopoulos E. (2011a) "Metaplastic Rocking Response of 1-dof systems on Shallow-improved Sand: An experimental Investigation", *Journal of Earthquake Engineering (submitted for possible publication)*.

Apostolou M., Gazetas G., Garini E. (2007), Seismic response of slender rigid structures with foundation uplifting, *Soil Dynamics and Earthquake Engineering*, 27 (7), pp. 642–654.

Chatzigogos C.T., Pecker A., Salencon J. (2009), Macroelement modeling of shallow foundations, *Soil Dynamics and Earthquake Engineering*, Vol. 29, No. 6, pp. 765–781.

Gajan S., Kutter B.L. (2008), Capacity, settlement, and energy dissipation of shallow footings subjected to rocking, *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, 134 (8), pp 1129-1141.

Gelagoti F., Kourkoulis R., Anastasopoulos I., and Gazetas G. (2011a) "Rocking Isolation of Frame Structures Founded on Isolated Footings", Earthquake Engineering and Structural Dynamics (in print)

Gelagoti F., Kourkoulis R., Anastasopoulos I., and Gazetas G. (2011b), "Rocking–isolated Frame Structures : Margins of Safety against Toppling Collapse and Simplified Design Approach", Soil Dynamics and Earthquake Engineering (in print *doi:10.1016/j.soildyn.2011.08.008*)

Gerolymos N., Apostolou M., Gazetas G. (2005), Neural network analysis of overturning response under near-fault type excitation, *Earthquake Engineering and Engineering Vibration*, 4 (2), pp. 213-228.

Kawashima K., Nagai T., Sakellaraki D. (2007), Rocking Seismic Isolation of Bridges Supported by Spread Foundations, *Proc. of 2<sup>nd</sup> Japan-Greece Workshop on Seismic Design, Observation, and Retrofit of Foundations*, April 3-4, Tokyo, Japan, pp. 254–265.

Kutter B.L, Martin G., Hutchinson T.C., Harden C., Gajan S., Phalen J.D. (2003), *Status report on study of modeling of nonlinear cyclic load–deformation behavior of shallow foundations*, University of California, Davis, PEER Workshop; March.

Mergos, P. E., Kawashima, K. (2005). "Rocking isolation of a typical bridge pier on spread foundation." *Journal of Earthquake Engineering*, 9(2): pp 395-414.

Paolucci R., Shirato M., Yilmaz M.T. (2008), Seismic behaviour of shallow foundations: Shaking table experiments vs numerical modelling, *Earthquake Engineering and Structural Dynamics*, Vol. 37, pp. 577–595.

Pecker, A., & Pender, M.J., (2000), Earthquake Resistant Design of Foundations : New Construction, Invited paper, *GeoEng2000*, Vol 1, pp. 313-332.

Vintzilaiou E., Tassios T.P., Chronopoulos M. (2007) "Experimental validation of seismic code provisions for RC columns", Engineering Structures, 29, pp1153-1164

Yim C.S., Chopra A.K. (1984), "Earthquake response of structures with partial uplift on Winkler foundation", *Earthquake Engineering and Structural Dynamics*, Vol. 12, pp. 263-281.