Seismic Behavior of Batter Piles: Elastic Response

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Abstract: Several aspects of the seismic response of groups containing nonvertical piles are studied, including the lateral pile-head stiffnesses, the "kinematic" pile deformation, and the "inertial" soil-pile-structure response. A key goal is to explore the conditions under which the presence of batter piles is beneficial, indifferent, or detrimental. Parametric analyses are carried out using three-dimensional finite-element modeling, assuming elastic behavior of soil, piles, and superstructure. The model is first used to obtain the lateral stiffnesses of single batter piles and to show that its results converge to the available solutions from the literature. Then, real accelerograms covering a broad range of frequency characteristics are employed as base excitation of simple fixed-head two-pile group configurations, embedded in homogeneous, inhomogeneous, and layered soil profiles, while supporting very tall or very short structures. Five pile inclinations are considered while the corresponding vertical-pile group results serve as reference. It is found that in purely kinematic seismic loading, batter piles tend to confirm their negative reputation, as had also been found recently for a group subjected to static horizontal ground deformation. However, the total (kinematic plus inertial) response of structural systems founded on groups of batter piles offers many reasons for optimism. Batter piles may indeed be beneficial (or detrimental) depending on, among other parameters, the relative size of the overturning moment versus the shear force transmitted onto them from the superstructure.

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Introduction

Batter piles have been used for a long time to resist large lateral loads from winds, water waves, soil pressures, and impacts. Their distinct advantage over vertical piles is that they transmit the applied lateral loads partly in axial compression, rather than only through shear and bending. Thus, batter piles offer larger stiffness and bearing capacity than same-diameter-and-depth vertical piles—a superiority of particular importance when the near-surface soils are soft and/or the lateral load is large.

Despite these advantages, they do not enjoy a good reputation for seismic resistance. Following the poor performance of batter piles in a series of earthquakes, the seismic behavior of inclined piles has been considered detrimental, and many codes require that such piles be avoided. For instance, the French Seismic Code (AFPS 1990) states flatly that "*Inclined piles should not be used to resist seismic loads.*" The seismic Eurocode EC8/Part 5, dealing with geotechnics and foundations, is a little less restrictive, stating: "*It is recommended that no inclined piles be used for transmitting lateral loads to the soil. If, in any case, such piles are used, they must be designed to carry safely axial as well as bending loading.*"

The main arguments that have been frequently mentioned by engineers as the real or perceived drawbacks of inclined piles include but are not limited to: (1) "parasitic" bending stresses due to soil settlement (following an earthquake) and/or soil consolidation (before the earthquake); (2) large forces (of alternating sign) onto the pile cap; (3) reduction in bending moment capacity due to seismically induced tensile forces; (4) undesirable permanent rotation of the cap when the inclination of the piles is not symmetric; and (5) increased structural shear due to the stiffening of the system.

Case histories that have recently confirmed the potential for unsatisfactory performance of improperly designed batter piles include the wharf in the Port of Oakland in the 1989 Loma Prieta earthquake (M_s=7.1), the Port of Los Angeles in the 1994 Northridge earthquake (M_s=6.8), and the Rio Banano and the Rio Vizcaya Bridges in the 1991 Costa Rica earthquake (M_s=7.5). The bad reputation of batter piles has been reinforced by these incidents. The culmination was the following statement in the ASCE monograph on "Seismic Design of Port and Harbor Facilities": "*The use of batter piles in ports is typically not encouraged because of their poor seismic performance during past earthquakes*" (Le Val Lund 2003; Kavazanjian 2006).

However, a more thorough investigation on the causes of these failures, showed that the inadequate reinforcement in the top of the piles and also the improper connection of piles to their caps were the culprits of the observed damage (Mitchell et al. 1991;

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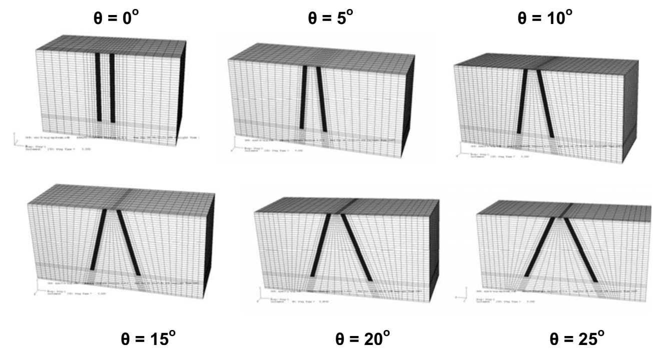

Fig. 1. Six studied pile group configurations and the corresponding 3D finite-element discretization

Priestley et al. 1991)—a result of the early "isostatic" method of analysis which assumed that batter piles transmit only axial load.

What if batter piles were properly designed to resist the developed moment and shear loads at their head? Furthermore, if they were designed to posses sufficient ductility at the head and the connection to the cap, would their seismic behavior still remain poor?

A goal of this paper is to give at least a partial answer to some of these questions. Indeed, in recent years, evidence has been accumulating that well-designed batter piles may not only have a satisfactory performance themselves, but may also be beneficial for the structure they support. Recent research on the seismic response of batter piles and micropiles (Guin 1997; Lam and Martin 1986; Sadek and Shahrour 2004, 2006; Gerolymos et al. 2008; Padron et al. 2009) has shown that the seismic response of a structure may improve in many respects when supported by inclined piles. Moreover, case histories referring to the Maya Warf in the Kobe 1995 earthquake and the Landing Road Bridge in the Edgecumbe, New Zealand 1987 earthquake have highlighted the potential help provided by inclined piles (Berrill et al. 2001; Gazetas and Mylonakis 1998).

As a result of the improved understanding of the source of the observed poor performance, batter piles in recent years seem to have been reestablished in their traditional role of withstanding large horizontal loads applied to deep foundations (as pointed out in an enlightening professional article by Kavazanjian 2006). The piers for the new San Francisco Bay Bridge East Span present a characteristic example of this trust in batter piles to carry huge lateral seismic loads in very soft soil.

Presently, research on the seismic response of batter piles has been rather limited (Juran et al. 2001; Sadek and Shahrour 2004, 2006; Okawa et al. 2005; Poulos 2006; Deng et al. 2007; Ravazi et al. 2007; Gerolymos et al. 2008). Aiming at filling part of this gap, we study several aspects of the seismic response of batter piles through parametric three-dimensional (3D) analyses employing the finite-element (FE) method. Only the idealized case of linear viscoelastic soil response is treated here. The shortcomings of linearity will be explored in a forthcoming companion paper, but it can be persuasively argued that the conclusions drawn in this study remain at least qualitatively valid even in the presence of soil nonlinearities; here are two reasons:

1. The results are presented here only in terms of *ratios* of response variables of the batter pile system with respect to the corresponding vertical-pile system (the response variables are: bending moments, axial and shear forces in the piles, displacement and rotation of the pile cap, and displacement of the structure). Using experimental results in a centrifuge, we show in the last section of the paper that such *ratios* are almost indifferent to soil nonlinearities, although of course the absolute values of each response quantity are particularly sensitive to the unavoidable near-surface inelasticity of the soil and geometric nonlinearity of the soil-pile interface.
2. One of the four types of idealized soil profiles chosen for our study has a shear modulus proportional to depth ($E=\lambda z$). For laterally loaded piles this profile may indirectly reflect soil nonlinearity. Because the values of the soil *secant shear modulus* near the surface of even a homogeneous deposit are likely to be drastically reduced (from their larger $G_{\max}$ values) due to the developing large shear and normal strains associated with large pile-head defections (see Velez et al. 1983).

The first part of the paper outlines the numerical model and shows its consistency with available analytical results.

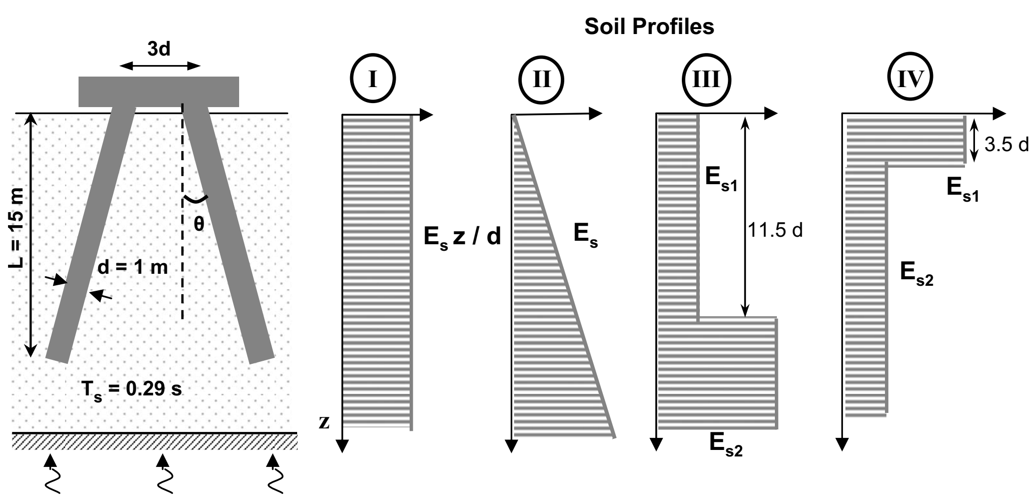

Fig. 2. Problem geometry and the four soil profiles

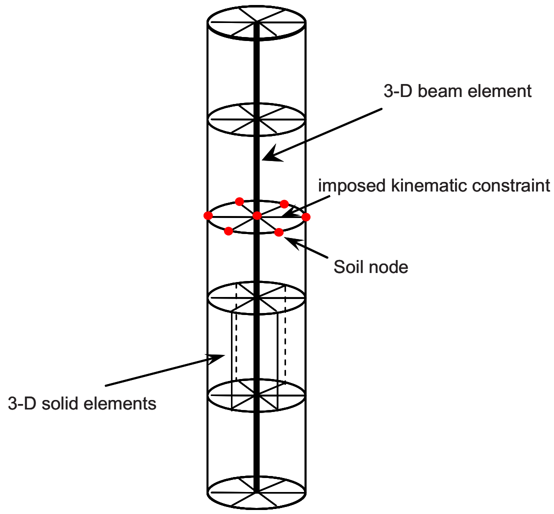

Fig. 3. Pile modeling: the piles are represented with a series of beam elements rigidly linked to the peripheral (soil) nodes in order to properly model the pile geometry. In this way, each pile section behaves as a rigid disk (rotation is allowed on the condition that the disk remains perpendicular to the beam axis, but stretching is prohibited) in a manner equivalent to that according to beam theory.

Problem Definition and Finite-Element Modeling

Model Description

The seismic behavior of symmetric 2×1 group configurations with piles battered at various angles is investigated using ABAQUS. Batter angles commonly encountered in practice are considered, such as 5–15°, in addition to the less usual cases of 20 and 25°. The vertical fixed-head pile group is used as a reference for delineating the role of pile inclination. Fig. 1 depicts the finite-element meshes of the six configurations. All piles are of Young's modulus $E_P=30$ GPa, diameter $d=1$ m, and the depth to their tip is $L=15$ m. The center-to-center distance, s, between the piles at pile-head elevation is three pile diameters ($s=3d$). The piles are rigidly connected to a perfectly rigid massless pile cap which is not in contact with the surrounding soil. The mass-and-column superstructure is modeled as a single degree of freedom oscillator. The concentrated superstructure mass M_{str} is such that the load per pile in each configuration is 1.0 MN, typical of actual pile designs. In all cases studied, the fixed-base fundamental period of the superstructure is $T_{\mathrm{str}}=0.44$ s and of the soil $T_{\mathrm{soil}}=0.29$ s.

Four idealized soil profiles are considered: (1) a homogenous; (2) a nonhomogeneous "Gibson" soil; (3) a two-layer profile with a bottom stiffer layer; and (4) a two-layer profile with a top stiffer layer (crust) (Fig. 2).

Modeling Assumptions and Simplifications

Both pile and soil are linear viscoelastic. Soil is modeled with eight-noded brick elements while the piles are represented with a series of 3D Euler-Bernoulli beam elements. The connection of the beam nodes with the corresponding peripheral soil nodes is established through appropriate kinematic constraints in order to properly model the pile geometry (Fig. 3). In this way, each pile section behaves as a rigid disk: rotation is allowed on the condition that the disk remains always perpendicular to the beam axis, but stretching cannot occur. Finally, full-bonding conditions are assumed at pile-soil interface: clearly a simplification of reality, but one which probably affects vertical and batter piles to a similar degree.

The performed mesh sensitivity study revealed that an element dimension of 0.5 m (i.e., one pile radius) leads to nearly accurate results. In the case of seismic loading appropriate kinematic constraints are imposed to the lateral edges of the model, allowing it to move in horizontal shear as the free field. For the inertial loading imposed on the pile head from the superstructure, "elementary" transmitting boundaries (ρV dashpots in all three directions) absorb much of the wave energy emitted from the oscillating piles. Note also that with the fixed-base fundamental period of the superstructure (0.44 s) being larger than the first natural period of the soil (0.29 s), spurious oscillations at the boundaries of the model are limited as a result of a *destructive interference* (existence of a cut-off period for radiation damping equal to the first natural period of the soil profile) of the outward spreading waves (Gerolymos and Gazetas 2006).

Three real acceleration time histories (Fig. 4), covering a wide range of frequencies, are used as seismic excitation at the base ("within" motion) of the finite-element model: (1) the record of

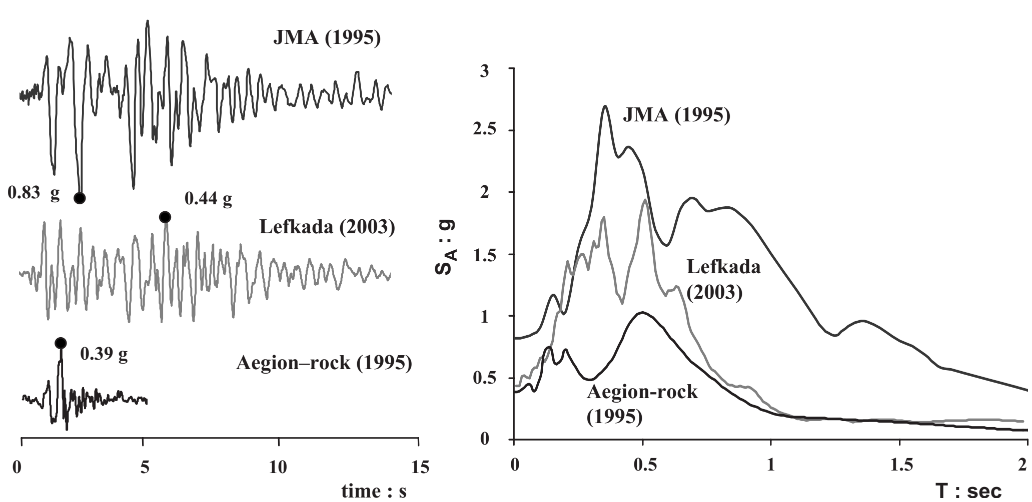

Fig. 4. Acceleration time histories used as base excitation and the corresponding 5% damped response spectra

the 2003 Lefkada M_s 6.4 earthquake: peak ground acceleration (PGA)=0.42g, dominant period range $T_p \approx 0.2-0.65$ s (Gazetas et al. 2005); (2) the rock outcrop motion of the 1995 Aegion M_s 6.2 earthquake: PGA=0.39g, dominant period range $T_p \approx 0.14-0.6$ s (Gazetas 1996); and (3) the Japan Mountaineering Association (JMA) record of the 1995 M_{JMA}=7.2 Kobe earthquake PGA=0.83g, $T_p \approx 0.25-1.0$ s. Note, however, that in view of the assumed elasticity, it is only the *frequency content* and the sequence of pulses of the records that mater, not the value of PGA. Our results are always in dimensionless form (i.e., ratio of responses) and hence are not affected by the value of PGA.

Static Stiffnesses, Convergence to Published Solutions

As a starting point, the static stiffnesses of a single batter pile are computed and contrasted with available solutions for vertical (Poulos and Davis 1980; Gazetas 1991) and batter (Poulos 1980) piles in Figs. 5 and 6. For a homogeneous soil stratum of thickness H, which exceeds the depth L to the pile tip (Soil Model I), our numerical (FE) results are compared with those computed: (1) on the basis of Poulos (1980) and Poulos and Davis (1980) approximation for angles of batter $\theta=0$ and 25° and (2) the approximate closed-form expressions of Gazetas (1991) for a vertical pile. The latter were developed using the Blaney et al. (1976) innovative dynamic finite-element which incorporates perfect transmitting boundaries. Fig. 5 depicts the variation of the normalized lateral static stiffnesses, K_{HH}, K_{RR}, K_{HR}, as functions of the ratio E_p/E_s of the pile and soil Young's moduli, and the angle of batter θ as a parameter. We draw the following conclusions:

- The rocking K_{RR} and, especially, the cross-coupled horizontal-rocking K_{HR} stiffnesses are essentially independent of θ: vertical and battered piles have nearly identical response, as expected by Poulos' 1980 simplification (Poulos and Madhav 1971; Poulos and Davis 1980). Therefore the closed-form expressions developed for vertical piles (Gazetas 1991)

$$K_{RR} \approx 0.15 d^3 E_S (E_P/E_S)^{0.75} \tag{1}$$

$$K_{HR} \approx -0.22 d^2 E_S (E_P/E_S)^{0.50} \tag{2}$$

  are in excellent accord with both the FE results of this study and Poulos' approximation. The above expressions can therefore be used even with batter piles.
- The horizontal stiffness K_{HH} exhibits a small dependence on θ. Both the FE analysis and Poulos' approximation show that stiffness increases by about 30% on average when θ increases from 0 to 25°. In percentage, the difference declines with increasing E_P/E_S ratio.
- The results of the Poulos (1980) and Gazetas (1991) approximations only slightly underestimate the finite-element results. The following simple expression:

$$K_{HH}(\theta) \approx 1.08 E_S d (1+4\tan^2\theta)(E_p/E_S)^{0.21(1+\tan^2\theta)^{-1}} \tag{3}$$

  has been developed by fitting the FE results for batter piles. It will suffice in practical applications for any angle θ.

For the linearly inhomogeneous stratum (Gibson soil) with Young's modulus of the form

$$E(z) = E_S z/d \tag{4}$$

in which apparently E_S=modulus at one-diameter depth, similar conclusions can be drawn from Fig. 6:

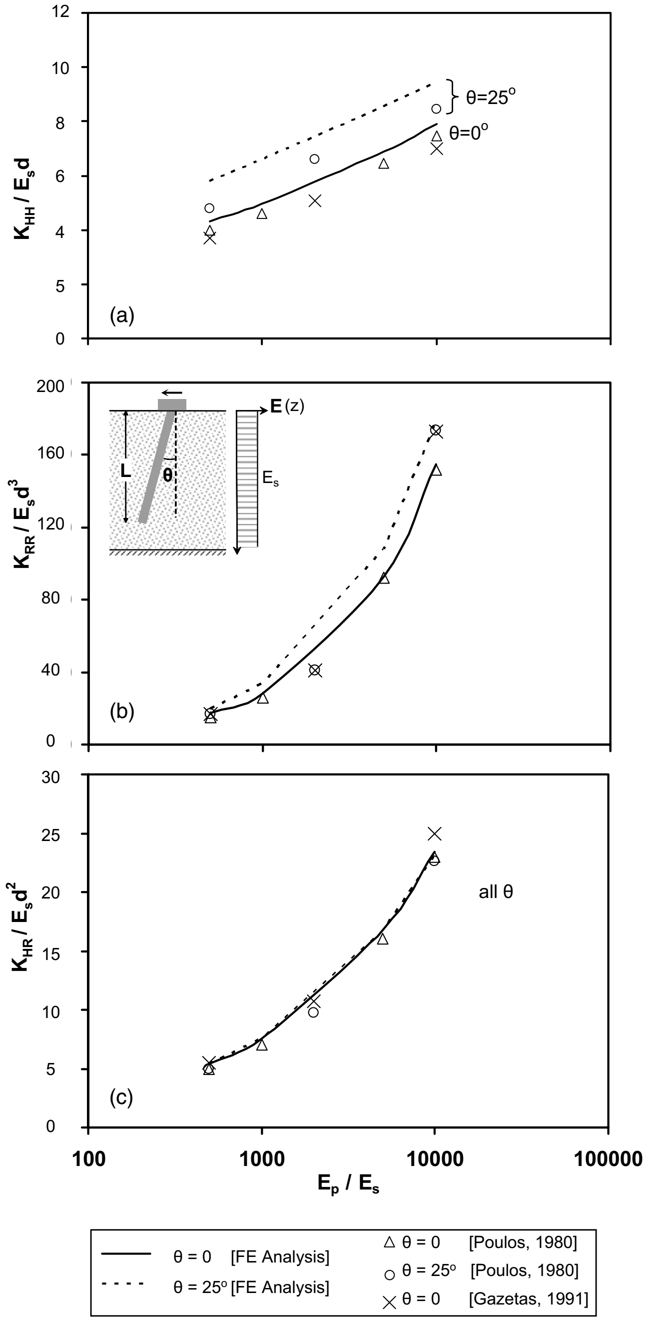

Fig. 5. Normalized static stiffnesses: (a) swaying; (b) rocking; and (c) cross swaying rocking for batter angle $\theta=0$ and 25° as a function of pile-soil stiffness ratio E_P/E_S (L/d=15, homogenous soil). Comparison with solution for vertical and inclined piles from the literature.

- The rocking and cross swaying-rocking stiffnesses, K_{RR} and K_{HR}, are again practically unaffected by the inclination of the pile, while they are sensitive to the E_P/E_S ratio. The Poulos (1980) and Gazetas (1991) approximations are again in excellent accord with the present FE results. Thus the expressions developed for vertical piles

$$K_{RR} \approx 0.15 E_S d^3 (E_P/E_S)^{0.80} \tag{5}$$

$$K_{HR} \approx -0.17 E_S d^2 (E_P/E_S)^{0.60} \tag{6}$$

  provide very good estimates for all values of θ [the reader should notice that E_S, the modulus at depth $z=d$, in the above

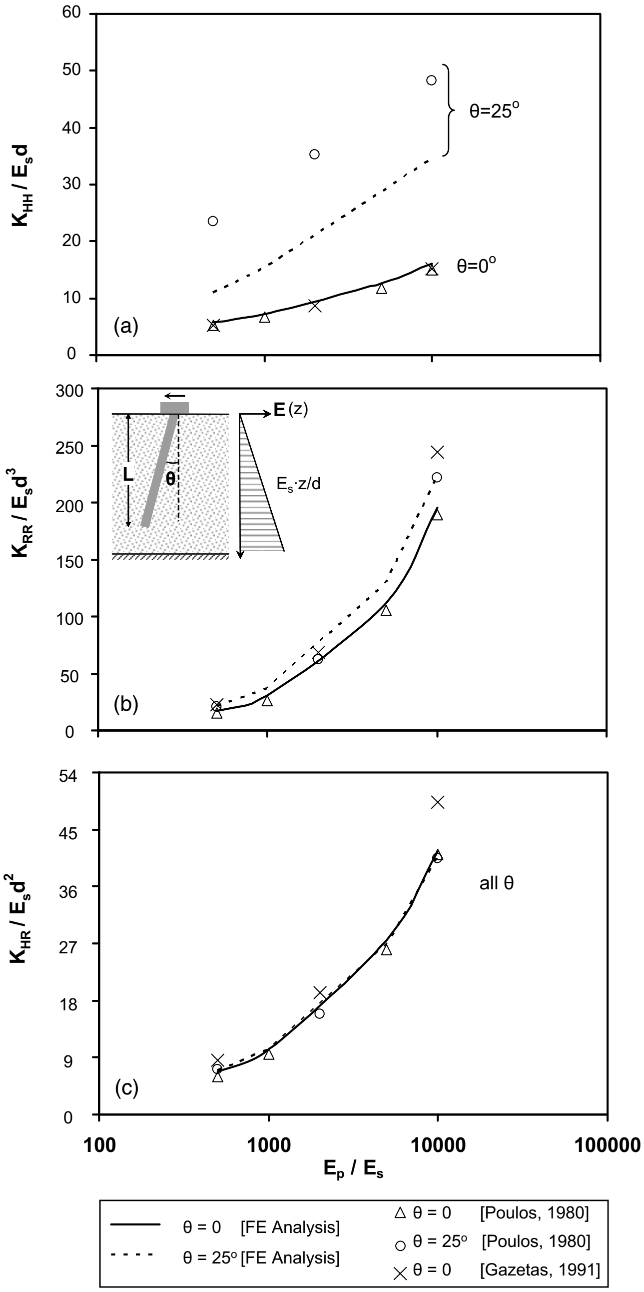

Fig. 6. Normalized static stiffnesses of single pile: (a) swaying; (b) rocking; and (c) cross swaying rocking for batter angles $\theta=0$ and 25° as a function of pile-soil stiffness ratio E_p/E_s ($L/d=15$, inhomogeneous Gibson soil). Comparison with solutions for vertical and inclined piles from the literature.

equations has a different meaning from the constant E_S modulus of Eqs. (1)–(3)].

- On the contrary, the horizontal stiffness K_{HH} is very sensitive to the batter angle θ. Increasing θ to 25° approximately doubles the stiffnesses according to our FE results, or triples them according to Poulos' approximation—for all values of the E_P/E_S ratio. We note that Poulos (1980) solution is based on an (additional) approximation, necessary to handle the soil inhomogeneity while still using Midlin's solution for a *homogeneous* half-space (Poulos 1979). This may be the cause of some inaccuracy which shows up with large pile inclinations.

The FE results for batter piles have been fitted with the expression

$$K_{HH}(\theta) \approx 0.60dE_S(1+\tan\theta)(E_P/E_S)^{0.35(1+0.5\tan^2\theta)^{-1}} \quad (7)$$

which applies to flexible piles of $L/d>10$. For $\theta=0$ it reduces to the vertical-pile expression of Gazetas (1991).

Note that all the above expressions [Eqs. (1)–(3) and (5)–(7)] apply only for floating piles. If the pile bears on a rigid base, the effect of θ becomes more prominent, noticeable even for K_{RR} and K_{HR}. It is also worth noting that the pile "slenderness" ratio, L/d, plays a more significant role with batter piles of large inclination than with vertical piles because the axial stiffness which affects K_{HH} of batter piles is more sensitive to pile length; while by contrast the stiffness K_{HH} of a vertical pile is essentially unaffected by L [at least for flexible pile; e.g., Randolph (1981); Banerjee and Davies (1978, 1980); Banerjee and Driscoll (1975, 1976); Poulos (1974, 1999)].

Kinematic Response of Inclined-Pile Groups

The kinematic response of vertical piles has been thoroughly studied by several researchers including Flores-Berrones and Whitman (1982), Kaynia and Kausel (1982), Dobry and O'Rourke (1983), Harada et al. (1981), Gazetas (1984), Fan et al. (1991), Kavvads and Gazetas (1993), Bentley and El Naggar (2000), Nikolaou et al. (2001), and Takewaki and Kishida (2005). However, little attention has been paid to the kinematic response of groups containing inclined piles. Among the few exceptions: Sadek and Shahrour (2006) studied the seismic response of inclined micropiles subjected to a sinusoidal motion at the eigenfrequency of the soil profile, and showed that for kinematic loading a group of four symmetrically inclined micropiles exhibits lower values of lateral acceleration at the cap level and larger values of internal forces in the piles compared to a group of vertical micropiles. Deng et al. (2007) performed kinematic analysis for a large pile group containing inclined piles and found that kinematic loading can have a major impact on the magnitude of the maximum axial force that develops in the batter piles. In their study, such piles developed five to eight times greater axial forces than the vertical piles.

Figs. 6 and 7 present selected results for a rigidly capped group of two fixed-at-the-cap symmetrically inclined piles, in Gibson soil. Detailed results are given in Fig. 6 only for the Lefkada-2003 excitation; the observed trends and conclusions, however, have been found to be valid for all earthquake motions (Giannakou 2007).

Distributions of displacements and internal forces (bending moment and axial force) along the piles at the time when the peak maximum values occur, normalized with the peak maximum value of the corresponding vertical-pile group, are presented in Fig. 7. The maximum values of the bending moments that develop for all motions and all soil profiles is summarized in Fig. 8. Several trends are worthy of note.

One advantage of groups with batter piles is the reduction of the lateral displacement at the pile cap [Fig. 7(a)]. Evidently, the incompatibility between free-field and inclined-pile displacement profiles becomes more pronounced as the inclination increases. As indicated by the slope of the displacement curves for $z=0$, a profound effect of increased angle of batter is the increase and, more importantly, the change in direction of the pile-head rotation. In other words, for the soil and pile group moving to the

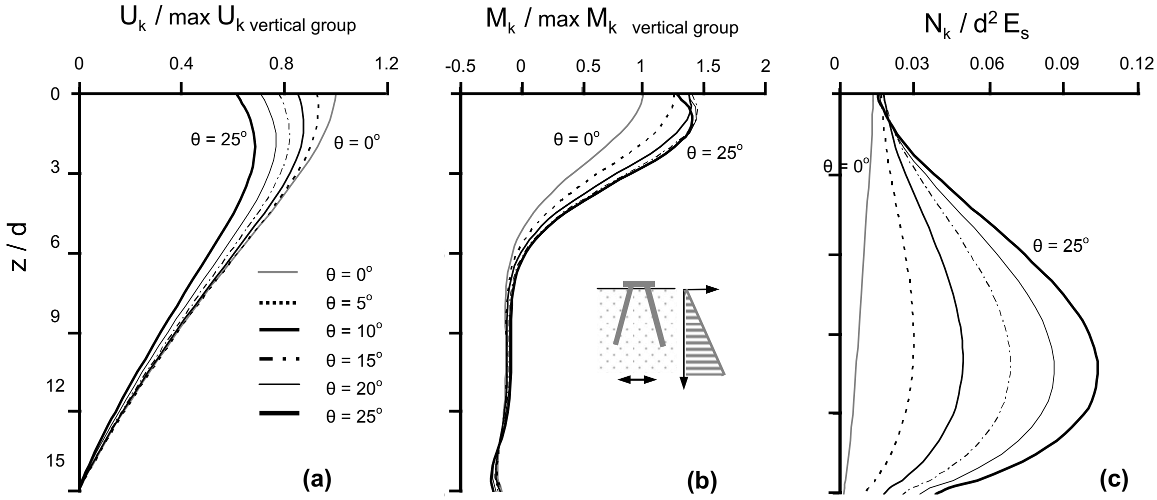

Fig. 7. *Kinematic* response of rigidly capped two-pile group: distributions of (a) horizontal displacement (relative to the displacement of the pile tip); (b) bending moment; and (c) axial force along the pile, for various pile inclination angles (Gibson soil, excitation: Lefkada record, T_{soil} $=0.29$ s, $E_p/E_s=1{,}000$, $L/d=15$)

right, Fig. 7(a) clearly reveals that the cap rotates counterclockwise.

It is interesting to note that the above conclusions are similar to those recently presented by Poulos (2006). Applying to a group of six rigidly capped piles a triangular horizontal ground displacement (with the maximum at the surface) he also found that "the group rotation is profoundly affected by the rake angle," and that for 15° angle of batter "the group rotation is more than four times greater than, and in the opposite direction to that for vertical piles" (Poulos 2006, p. 799).

The bending moments and axial forces that develop in fixed-head pile groups containing batter piles are larger than in the vertical group, in the case of the Gibson soil as depicted in Figs. 7(b and c). Notice that the axial force is normalized with the product d^2 E_S, where E_S=Young's modulus at depth $z=d$=pile diameter (since the axial force in the vertical piles is negligibly small). Evidently, kinematic interaction has a major effect on the maximum seismic load of batter piles. While the horizontal motion of the soil during the passage of seismic waves tends to cause mainly lateral motion of vertical piles (and thus they develop

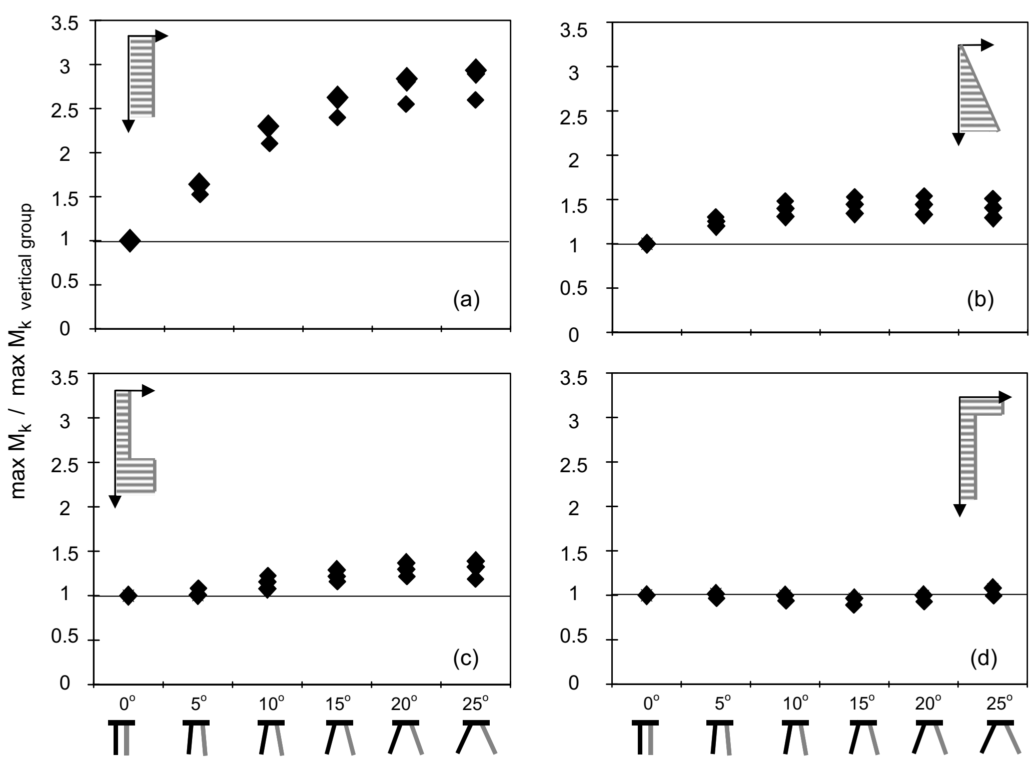

Fig. 8. *Kinematic* response of rigidly capped two-pile group: normalized peak maximum bending moment along the pile. Normalization with respect to the response of the group of vertical piles [maximum, minimum, and average values from the three accelerograms (Lefkada, Aegion, JMA)]. The results refer to the configuration and piles shown in gray at the bottom of the figure (T_{soil}=0.29 s).

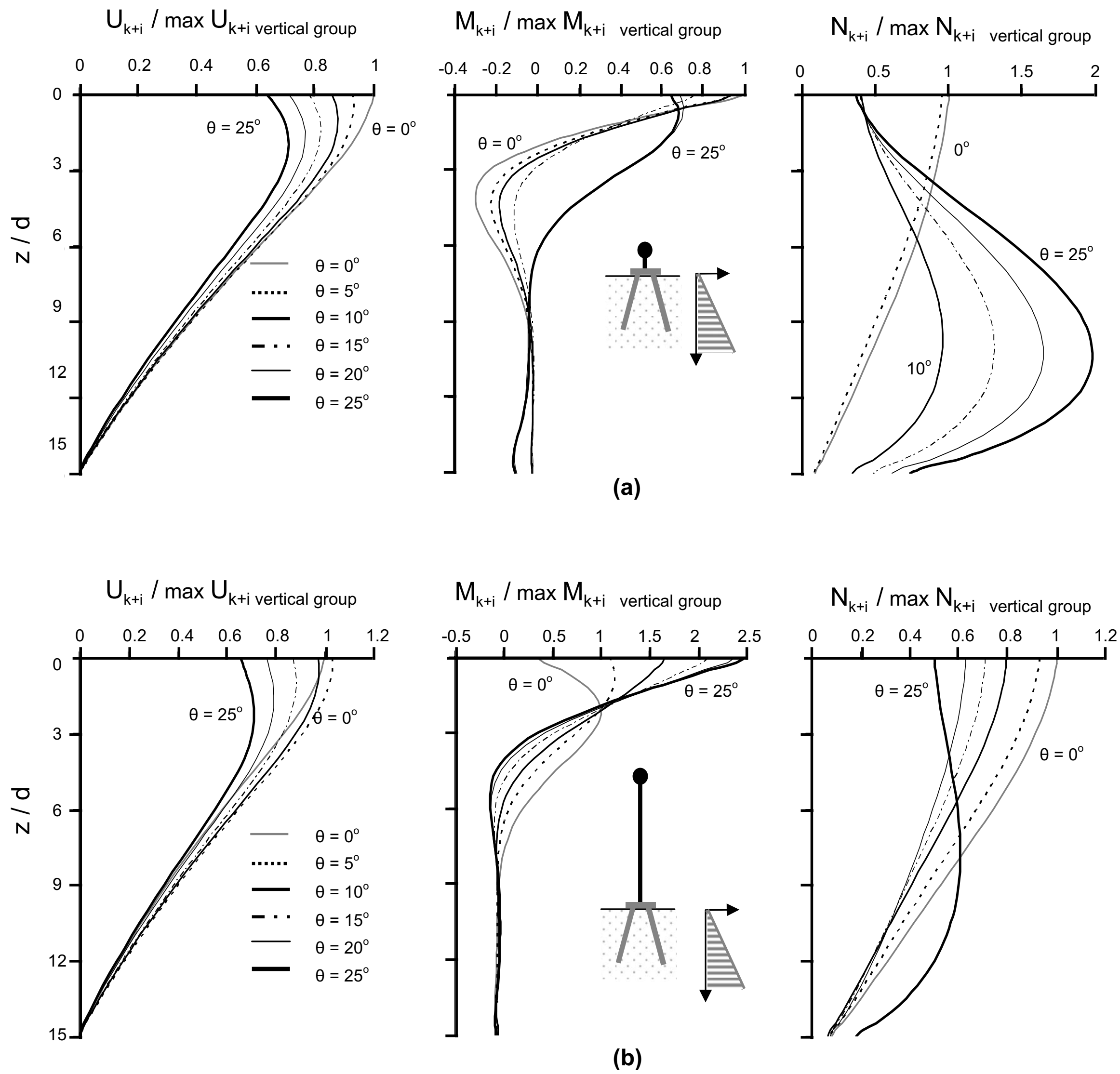

Fig. 9. Total (kinematic+inertial) response: distributions of horizontal displacement, bending moment, and axial force along the pile supporting (a) a short superstructure; (b) a tall superstructure (Gibson soil, excitation: Lefkada record, $T_{\text{soil}}=0.29$ s, $E_p/E_s=1{,}000$, $T_{\text{str}}=0.44$ s)

mainly shear and moment), inclined piles experience significant axial forces as well.

These effects are largest in the homogeneous soil profile. However, in profiles where a stiff soil layer is present (i.e., III and IV) the maximum bending moment that develops does not vary significantly with pile inclination [Figs. 8(c and d)].

In order to understand these differences among the developing moments in the four soil profiles, we need to consider the two sources of *kinematic* straining of batter piles. The first and most obvious cause is the existence of an abrupt change in stiffness between two successive soil layers (profiles III and IV). In this case the largest bending moment is generated at or near the interface of the two layers as is well known from the studies on vertical piles (Dobry and O'Rourke 1983; Nikolaou and Gazetas 1997), and hence it is practically independent of pile inclination [note that this source of kinematic straining of deep foundations is explicitly recognized in some recent codes (e.g., EC8)].

The second, and perhaps more important, source of kinematic straining is the constraint imposed by the rigid pile cap. In this case the maximum kinematic bending moment develops at or near the pile head (and will be "later" added to the inertial bending moment generated by the oscillation of the superstructure), in contrast to the aforementioned case where the maximum bending moment is generally developed at greater depths. In the case of the homogenous soil, the E_P/E_S ratio near the soil surface is relatively small; therefore the increase of batter angle leads to significantly larger values of bending moment due to the increase in lateral stiffness. In case of Gibson soil, however, the E_P/E_S ratio near the soil surface is very large, and the additional stiffness provided by the pile inclination leads to smaller increase in bending moments (in the order of 1.5 times the bending moment in the vertical pile compared to 3 in the case of the homogenous soil).

Soil-Pile-Structure Interaction

The influence of the (super)structure on the seismic response of groups with batter piles is considered in this section. Specifically, to illustrate the effect of structural height, two one-degree-of-freedom oscillators are studied, modeling: (1) a tall slender structure ($H_{st}=12$ m) whose crucial loading is the overturning

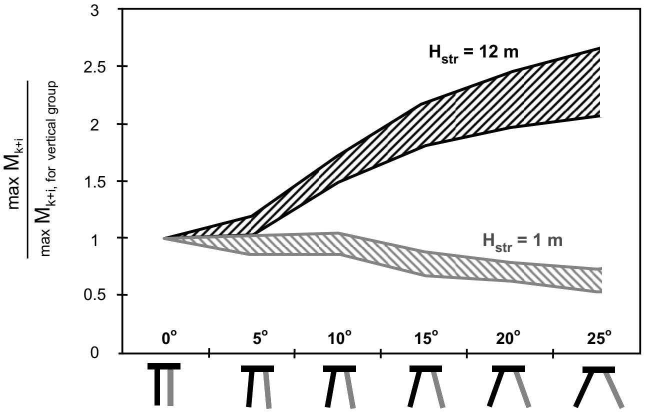

Fig. 10. Total (kinematic + inertial) response: normalized peak maximum bending moment along the pile supporting a short (H_{str}=1 m) and a tall (H_{str}=12 m) structure. Normalization with respect to the response of the group of vertical piles [maximum, minimum, and average values from the three accelerograms (Lefkada, Aegion, JMA)]. The results refer to the configuration and piles shown (in gray) at the bottom of the figure (Gibson soil, T_s=0.29 s, E_p/E_s =1,000, T_{str}=0.44 s).

moment, an example being a tall bridge pier and (2) a short structure (H_{str}=1 m) whose crucial loading is the shear force. The fundamental fixed-base period of the two structures is kept constant, 0.44 s. The groups are embedded in the nonhomogeneous Gibson-type profile (II).

Pile distress and the role of batter are highly dependent on the type of crucial loading (moment versus shear). This is shown in Figs. 9–11. Specifically, Fig. 9 presents the distributions of lateral displacement and internal forces (bending moment and axial force) that develop in the piles supporting a short [Fig. 9(a)] and a tall [Fig. 9(b)], structure at the time when the largest maximum occurs. Figs. 10 and 11 summarize in dimensionless form of the maximum bending moment and axial force that develop in the piles of the two structures, for all earthquake excitations. Note that groups with batter piles lead to small lateral displacement in earthquake shaking. Horizontal displacements decrease invariably for both structures as the inclination increases. However, groups with inclined piles develop larger cap rotations regardless of the type of structure they support!

The bending moment experienced by batter piles supporting a *tall* structure is larger than by vertical piles (Figs. 9 and 10). The bending moment increases monotonically with pile rake. This conclusion is valid even for large fixed-base fundamental structural periods where the inertial effect is limited. Proper reinforcement of the pile-cap connection is necessary for undertaking safely this bending moment and for securing adequate inelastic deformation in an unpredictably large earthquake motion. On the contrary, groups with inclined piles supporting a *short* structure develop smaller bending moments than the vertical group. This observation is in agreement with other published numerical and experimental studies (Sadek and Shahrour 2004, 2006; Okawa et al. 2005), and is also compatible with the prevailing engineering perception about the role of batter piles, as will be explained in the sequel. However, when the contribution to bending moment from inertial loading of the structure is small (i.e., flexible structures with very large fundamental periods), the contribution to bending moment from the kinematic deformation of the pile prevails and batter piles may suffer larger bending moments than vertical piles.

Perhaps surprisingly, groups of inclined piles supporting a tall structure attract smaller axial forces than those of the group with vertical piles (Fig. 11)! This must be attributed to the disproportionately large overturning moment resisted mainly by axial reactions of the vertical piles. However, batter piles embedded in relatively stiff soils (small E_P/E_S ratio) and supporting structures with large (fixed-base) fundamental periods may develop larger axial forces than the vertical piles owing to the larger contribution of the kinematic soil-pile interplay to their seismic response. In stark contrast, in the case of a short structure, inclined piles develop larger axial forces than the vertical piles. This maximum, however, occurs at great depths (in the order of 10 pile diameters) and is mainly due to the kinematic interaction of the pile with the soil [Figs. 9(a) and 7(c)].

To better understand the differences in the distress of batter piles supporting a tall and a short structure, we compare the snapshots of displacement of a vertical and a 25°-batter pile group, supporting either a *short* (H_{str}=1 m) and a *tall* (H_{str}=12 m) structure. Fig. 12 portrays the comparison. Notice that in the case of the batter group supporting a short structure [Fig. 11(a)] the displacement rotation of the pile cap is out-of-phase with the displacement of the structure. This is not the case with the tall structure: cap rotation and structure are nearly in-phase.

Figs. 13 and 14 attempt to illustrate schematically the mechanisms through which the inertial forces of the superstructure are undertaken. Our goal is to convince that the presented results are explainable (hence reasonable), and at the same time to develop a deeper understanding of the problem mechanics.

With the *short* structure, where the inertial *shear force* dominates, the vertical group develops primarily a pair of shear forces, and only secondarily bending moments due to the rotation fixity at the cap [Fig. 13(a)]. Axial forces at the vertical piles are negligible.

By contrast, a horizontal shear force on the cap of a batter pile group results in the development of both shear and axial forces in each pile. In fact, since the lateral pile deformation far exceeds the deformation due to the axial forces, the arrows of the shear-force vectors define qualitatively the rotation of the cap [Fig. 13(b)].

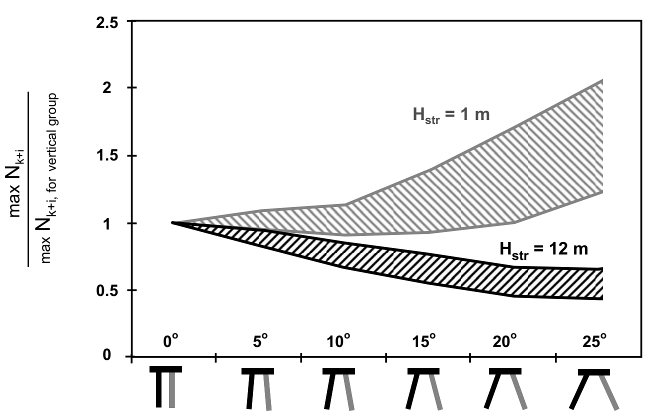

Fig. 11. Total (kinematic + inertial) response: normalized peak maximum axial force along the pile supporting a short (H_{str}=1 m) and a tall (H_{str}=12 m) structure. Normalization with respect to the response of the group of vertical piles [maximum, minimum, and average values from the three accelerograms (Lefkada, Aegion, JMA)]. The results refer to the configuration and piles shown (in gray) at the bottom of the figure (Gibson soil, T_{soil}=0.29 s, E_p/E_s=1,000, T_{str}=0.44 s).

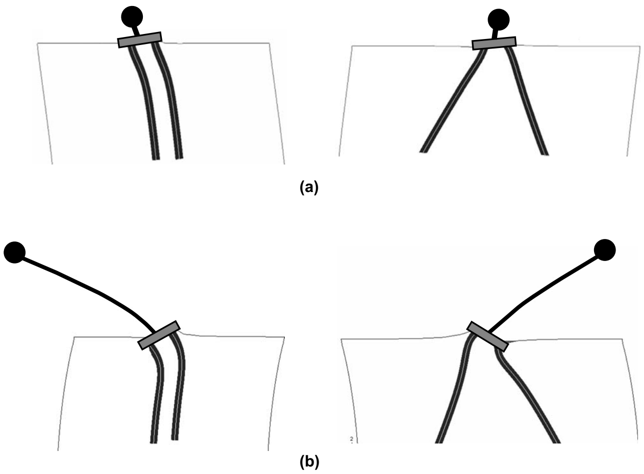

Fig. 12. Exaggerated snapshots of the deformed shapes of groups with vertical piles (left-hand side) and inclined (right-hand side) supporting: (a) short ($H_{str}=1$ m); (b) tall ($H_{str}=12$ m) superstructures in Gibson soil

With the tall structure, where the *overturning moment* dominates, the vertical piles are subjected to a pair of axial forces that undertake most of this load [Fig. 14(a)]. Secondarily, head moments develop due to pile fixity to the cap. In stark contrast, batter piles may undertake this large moment mainly largely by flexure (bending) [Fig. 14(b)]. As a result, substantial cap rotations take place. Indeed, the capped batter piles are now rotationally more flexible than the capped vertical piles! To elucidate why, imagine two piles inclined at very large (certainly unrealistic) angles (Fig. 15) with a large overturning moment applied on the cap: "replacing" conceptually the soil with Winkler springs, one can realize that the piles will unavoidably bend, activating these springs alternately in tension and compression, near the cap, q. The (remaining) portion of the load, undertaken by the framing action of the piles activates the friction, f, at the pile-soil interface, would be very small [equal to zero in the extreme (unrealistic, of course) case of two "piles" inclined at 90°].

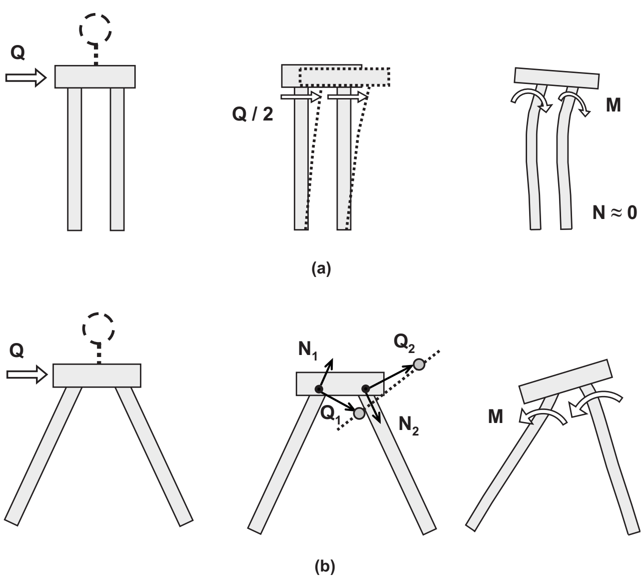

Fig. 13. Mechanisms for undertaking the inertial forces of a short superstructure for (a) vertical; (b) inclined-pile groups. *The vectors indicate the forces imposed from the on the piles*. The dashed lines correspond to the location of the cap if the axial displacements of the piles are completely ignored.

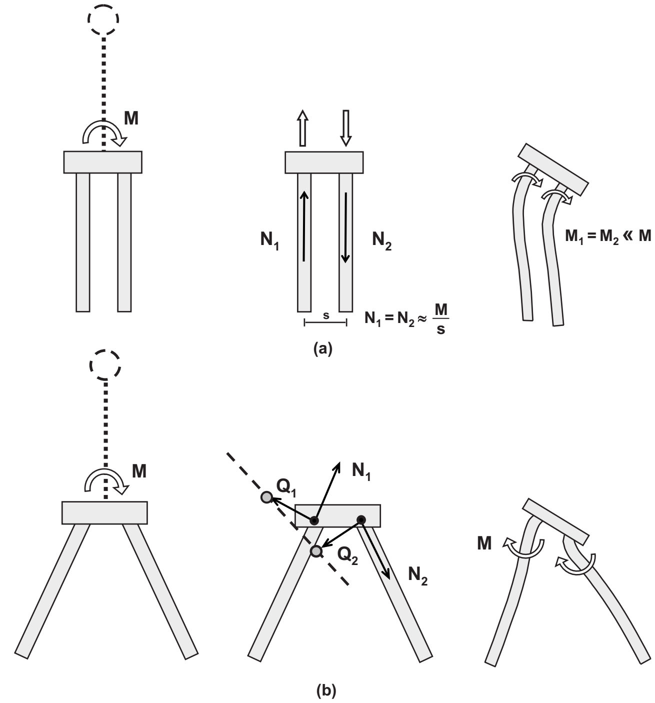

Fig. 14. Mechanisms for undertaking the inertial forces of a tall (slender) superstructure for (a) vertical; (b) inclined-pile group. *The vectors indicate the forces imposed from the cap on the piles*. The dashed lines correspond to the location of the cap if the axial displacements of the piles are completely ignored.

Having analyzed the mechanics of the pile distress, let us examine the structural distress. The so-called "floor" acceleration response spectra at the mass of the superstructure (i.e., the response spectra of the computed motion of the superstructure mass) normalized by the peak ground acceleration of the free field are depicted in Fig. 16 for the tall and the short structure. Dimensionless diagrams for the drift of the superstructure are presented in Fig. 17. The following observations are noteworthy:

- Spectral accelerations decrease as the pile inclination increases, for both structures. The effect, however, with few exceptions is not significant.
- The horizontal drift when the structure is supported on a group of batter piles is generally smaller than with the exclusively vertical-pile group. This reduction of structural distress is appreciable only with the tall structure, and is attributed to the

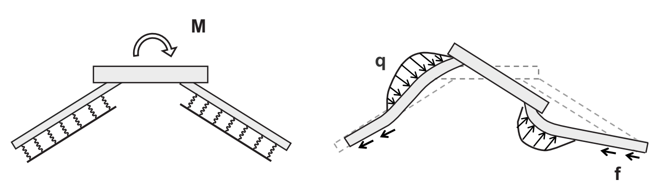

Fig. 15. Cartoon to elucidate the bending of piles with (grossly exaggerated) inclination, as they undertake the overturning moment transmitted from the superstructure primarily by flexure; in contrast with two vertical (at substantial distances) which will react mainly in axial compression and extension

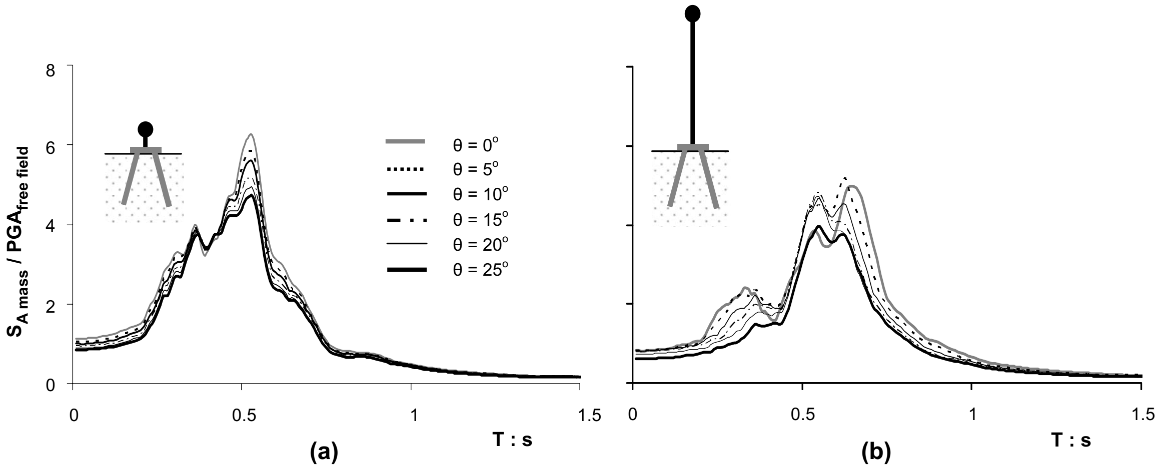

Fig. 16. Total (kinematic+inertial) response: normalized response spectra of the motion of the mass of (a) a short; (b) a tall superstructure. Normalization with respect to the free-field peak ground acceleration (Gibson soil, excitation: Lefkada motion, $T_{\mathrm{soil}}=0.29$ s, $E_p/E_s=1{,}000$, $T_{\mathrm{str}}=0.44$ s).

observed simultaneous occurrence *of smaller* horizontal cap displacement, and larger cap rotation with the group of batter piles. Increased pile batter relates to smaller structural distress and smaller horizontal displacement of the mass, at the cost of larger cap rotation.

Glimpse on the Possible Effects of Soil and Interface Nonlinearities

The two key limitations of our analyses are the assumed (1) linearity of soil behavior and (2) perfect contact at the soil-pile interface. Clearly, under strong seismic shaking and, especially, large inertia forces transmitted onto the foundation, the piles undergo lateral deflections that inevitably mobilize passive-type soil failure in front of the piles accompanied by gap formation and sliding in the back and the sides of the piles, respectively. There is no doubt that, in absolute terms, a linear full-contact analysis as the one used in this article cannot possibly capture such strong (geometric and material) nonlinearities. The explicitly stated presumption of the paper is that such nonlinearities affect the two types of pile foundations, i.e., with the vertical and with the batter piles, to about the same degree. So that the *ratio* of the respective responses, i.e., the "batter" response divided by the "vertical" response, is not particularly sensitive, if not indifferent, to the soil and interface nonlinearities.

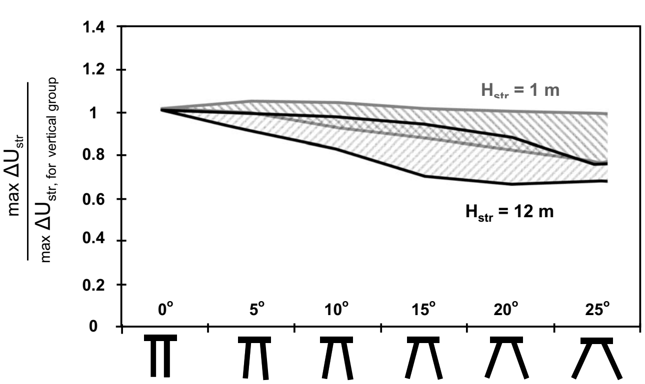

Fig. 17. Total (kinematic+inertial) response: normalized peak maximum horizontal drift of a short ($H_{\mathrm{str}}=1$ m) and a tall ($H_{\mathrm{str}}=12$ m) structure. Normalization with respect to the response of the group of vertical piles [maximum, minimum, and average values from the three accelerograms (Lefkada, Aegion, JMA)] (Gibson soil, $T_{\mathrm{soil}}=0.29$ s, $E_p/E_s=1{,}000$, $T_{\mathrm{str}}=0.44$ s).

To show that the above hypothesis is reasonable we utilize herein the results of a series of centrifuge tests recently performed in Japan (Tazoh et al. 2009). Fig. 18 depicts the centrifuge model of the two groups, consisting of 2×2 vertical and 2×2 batter piles (10° inclination), respectively. The piles are embedded in dry sand of 60% relative density that was subjected to a number of idealized and actual recorded time histories of base excitation. For the El Centro 1940 record, scaled to peak ground acceleration values of 50, 100, and 200 gal., the bending strain, ε_M, and axial strain, ε_A, histories at the pile head were recorded. From their Fourier spectra, $\varepsilon_M(f)$ and $\varepsilon_A(f)$, the corresponding transfer functions $\varepsilon_M(f)/A(f)$ and $\varepsilon_A(f)/A(f)$ were obtained, where $A(f)$ the Fourier amplitude of the excitation, function of frequency f. They are plotted in Fig. 19. The effect of nonlinearity is evident in the transfer functions: not only does the fundamental frequency decrease with increasing A, but with the intensity of 200 gal. (a particularly strong shaking for a $Dr=60\%$ sand) the fundamental resonance has been replaced by a much flatter and lower-amplitude spectrum both for bending and axial strain. Nevertheless, despite these individual differences the ratios of batter to vertical bending and axial strain transfer functions, plotted in Fig. 20 for the three levels of PGA, are indeed nearly independent of the intensity of shaking, and hence of at least moderate degrees of soil and interface nonlinearity.

This experimental evidence provides support to the claim that the results of the paper offer a reasonable approximation to reality. But, of course, the need for (additional) nonlinear analyses remains.

Conclusions

1. The purely *kinematic* response of batter piles tends to confirm their negative reputation: the parametric analyses show that they experience larger bending moments than vertical

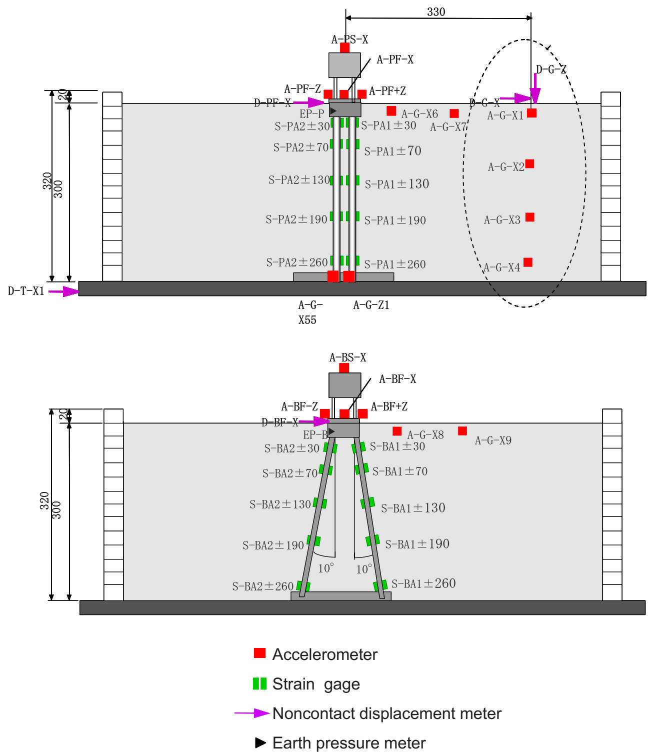

Fig. 18. Sections of the experimental setup of the centrifuge tests (dimensions in mm). A 2×2 group of vertical piles and a 2×2 group of inclined piles battered at 10°, embedded in dry sand of 60% relative density that were subjected to a number of idealized and actual recorded time histories of base excitation.

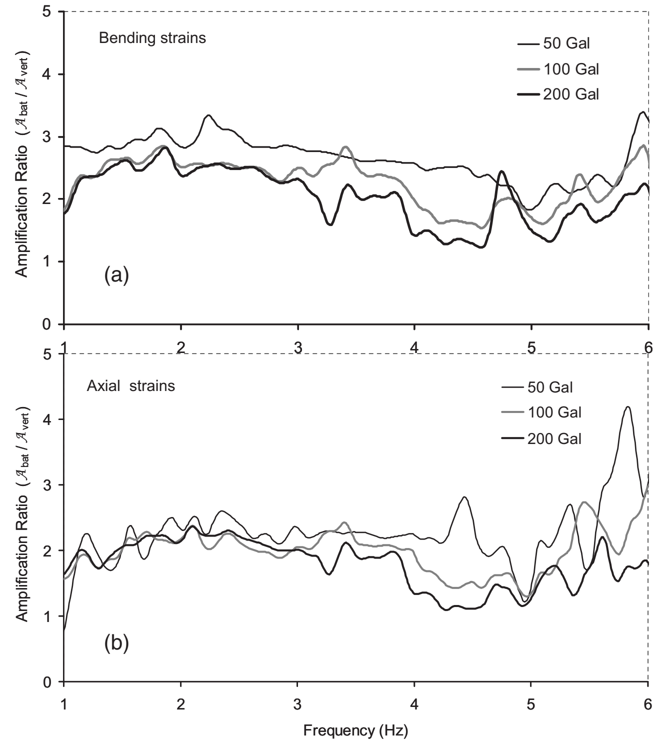

Fig. 20. Ratios of batter to vertical transfer functions of (a) bending; (b) axial strains at the pile head, with respect to the El Centro acceleration time history scaled to peak ground acceleration of 50, 100, and 200 gal.

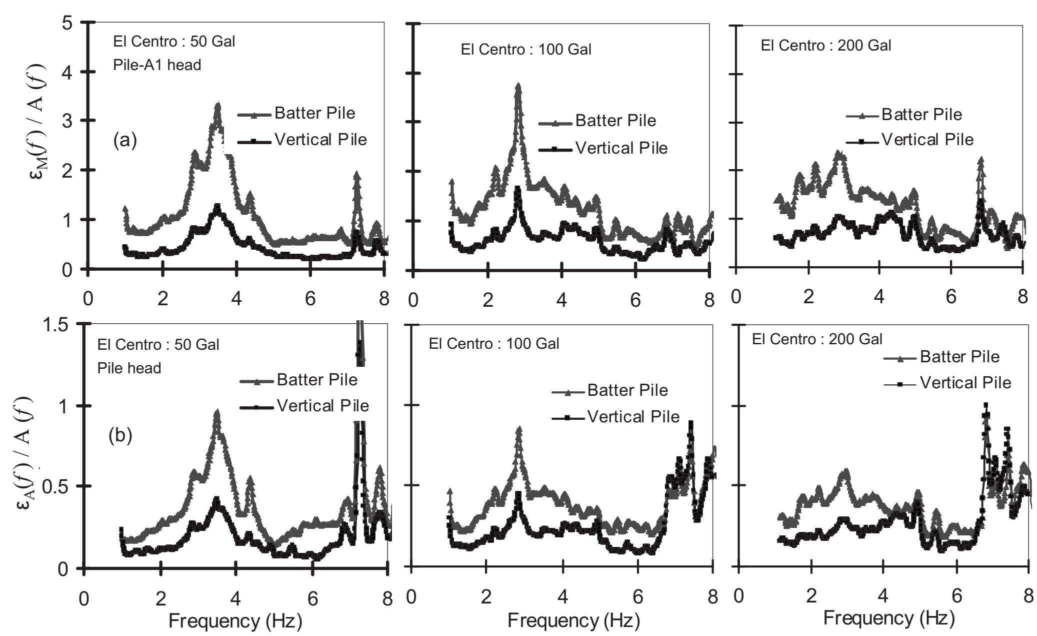

Fig. 19. Transfer functions of (a) bending strain; (b) axial strain at the pile head with respect to the El Centro acceleration time history as base excitation, for the two centrifuge models (2×2 batter piles—gray lines with triangles; 2×2 vertical piles—black lines with squares). Three levels of peak ground acceleration have been considered: 50, 100, and 200 gal.

piles. Moreover, batter piles exhibit significantly larger axial forces than vertical piles for all four idealized profiles, due to exclusively horizontal shaking of the soil. In fact both of these internal forces increase as the angle increases. This conclusion is in full accord with the conclusion of a recent study by Poulos (2006) who imposed on the piles a lateral static ground displacement linearly decreasing with depth, and thus not very different in shape from the first mode free-field displacements that is the main source of kinematic pile straining in our study.

2. However, the total (kinematic plus inertial) response of structural systems founded on groups of batter piles offers many reasons for optimism. It has been shown that the role of batter may be quite beneficial or detrimental depending on the relation between shear force and overturning moment. Specifically, a tall (slender) structure and a short (squatty) structure have been selected, as two extreme cases: "large" moment and "small" shear characterizes the tall structure; large shear and small moment the short one.
3. For the batter piles supporting a *tall slender* structure we conclude that:
   a. Configurations with batter piles undergo smaller horizontal displacements than the vertical group, but at the same time they develop larger cap rotations, often out-of-phase with lateral displacements.
   b. The bending moment in batter piles is larger than in vertical piles. In fact, the bending moment increases as the pile rake increases. Proper reinforcement of the pile-cap connection is necessary for undertaking safely this bending moment and securing adequate inelastic deformation in case of an unpredictably large (exceeding the design) earthquake motion.
   c. Perhaps surprisingly, the symmetric group of batter piles attracts smaller axial forces than a group of exclusively vertical piles! This is attributed to the disproportionately large share of the overturning moment resisted by axial reactions in the vertical piles—not the case with batter piles which undertake this moment partially with flexure (bending).
   d. The lateral distortion (and drift) of the structure on batter piles is significantly smaller than of on vertical piles.
4. For batter piles supporting a *short* squatty structure:
   a. Embedded in Gibson soil they develop smaller bending moments than vertical piles, within the range of the considered excitations.
   b. Now batter piles sustain larger axial forces than the vertical group, for two reasons: (1) kinematic loading, which constitutes an important component of the *total* loading, produces larger head moment in the batter piles and (2) the inertial loading induces mostly a dynamic shear force which, while being resisted by lateral loads in vertical piles, it loads axially (and laterally) the batter piles.
   c. The horizontal drift of the superstructure is less sensitive to pile batter.

This paper has tried to contribute toward a better understanding of the seismic behavior of batter piles, which may under certain circumstances be beneficial rather than detrimental, for both the structure they support and the piles themselves. Admittedly, the linear approximation of the soil-structure interaction (SSI) phenomena is not without shortcomings. Phenomena created by strong nonlinearities of the soil, such as permanent soil deformations due to extensive soil plastification, residual bending moments on the piles etc., cannot be captured by linear (or equivalent linear) FE analyses. Nonetheless, valuable insight is gained into better understanding the behavior of batter piles.

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