

Seismic response of slender rigid structures with foundation uplifting

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Abstract

The rocking of rigid structures uplifting from their support under strong earthquake shaking is investigated. The structure is resting on the surface of either a rigid base or a linearly elastic continuum. A large-displacement approach is adopted to extract the governing equations of motion allowing for a rigorous calculation of the nonlinear response even under near-overturning conditions. Directivity-affected near-fault ground motions, idealized as Ricker wavelets or trigonometric pulses, are used as excitation. The conditions under which uplifting leads to large angles of rotation and eventually to overturning are investigated. A profoundly nonlinear rocking behavior is revealed for both rigid and elastic soil conditions. This geometrically nonlinear response is further amplified by unfavorable sequences of long-duration pulses in the excitation. Moreover, through the overturning response of a toppled tombstone, it is concluded that the practice of estimating ground accelerations from overturning observations is rather misleading and meaningless.

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1. Introduction

While slender structural systems with a shallow foundation are generally considered as bonded to the ground, during strong seismic shaking uplifting from the supporting soil is often practically unavoidable. Examples of structures that experienced uplifting from the supporting soil have been reported in numerous earthquakes, including that of Chile 1960, Alaska 1964, San Fernando 1971, Kocaeli 1999, and Athens 1999. It is well documented in the literature that uplifting changes the rocking behavior in a profoundly nonlinear sense and modifies the structural response in most cases favorably. Apart from civil structures, uplifting and overturning are some of the most familiar phenomena for free-standing bodies (such as appended equipment, furniture, etc.) during strong earthquakes.

Since the pioneering work of Milne and Perry in 1881 [1,2] the uplifting and overturning response of rigid bodies has attracted the interest of many earthquake engineers and seismologists for over a century [3]. Early analyti-

cal and experimental studies conducted mostly in Japan had been motivated by tombstones overturnings after large earthquakes (Sagisaka, Inouye, Kimoura, Ikegami among others [3]). Housner [4] investigated in detail the rocking behavior of rigid blocks subjected to base excitation. Using an energy approach he uncovered the role of the excitation frequency and of the block size on the overturning potential. Makris and his co-workers [5,6] focused on the transient response of rigid blocks under near-source ground shaking idealized as trigonometric pulses, and derived the acceleration amplitude needed for overturning. Ishiyama [7] studied the slide-rocking motion of a rigid body on rigid floor and established criteria for overturning. Psycharis [8] introduced the compliance of the supporting soil with a viscoelastic Winkler foundation, extracted the linearized equations of rocking motion, and addressed the structural response of an uplifting system. Koh et al. [9] extended Psycharis' work on the linearized rocking response on flexible foundation by introducing the flexibility of the superstructure. Huckelbridge and Clough [10] carried out $\frac{1}{3}$ -scale shaking table tests with 9-story steel moment frames and confirmed the beneficial role of transient uplift on structural response.

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In the present study, two different systems of structures undergoing rocking motion with uplift are examined (Fig. 1):

- a rigid block supported on *undeformable ground*, which will be referred to herein as “*rigid foundation*”;
- a rigid block founded on an elastically deformable continuum in the form of a *homogeneous halfspace* or a *stratum over rigid bedrock*.

The conditions under which uplifting of these simple systems leads to large angles of rotation and eventually to overturning are investigated, and minimum acceleration levels for overturning are derived. Ground motion is mainly represented with two drastically different records (despite their nearly identical PGAs and not very different strong motion duration) obtained in the Athens and Kocaeli earthquakes of 1999, as well as with idealized Ricker-wavelets and one-cycle sinusoidal pulses.

2. Uplifting and overturning on a rigid base

We consider first a rigid rectangular block with aspect ratio b/h (half width over half height ratio) simply supported on a rigid base, which is oscillating horizontally. The coefficient of friction is adequately large so that sliding is prevented. As long as the overturning moment ($ma_g h$) about the base edge (where a_g = the base acceleration) does not exceed the restoring moment (mgb), the block remains attached to the base and undergoes only horizontal oscillation. As soon as the restoring moment is exceeded uplifting occurs setting the block on rocking motion. The system configuration is illustrated in Fig. 1a. Under static conditions, once uplifting is initiated about the corner point, the body overturns. Thus, the critical uplifting

acceleration of the base is identical with the minimum required to statically overturn the block in units of g (acceleration of gravity)

$$a_{\text{over,stat}} = a_c = \frac{b}{h} \tag{1}$$

However, under dynamic base excitation the inertia force “quickly” changes direction as the acceleration changes sign, and overturning is avoided. Rocking oscillation takes place with the two corner points, O and O', being alternately the pivot points. Between two successive impacts the governing equation of rocking motion can be expressed in the compact form

$$\ddot{\theta}(t) = -p^2(\sin[\theta_c \text{sgn}\theta(t) - \theta(t)] + a_g \cos[\theta_c \text{sgn}\theta(t) - \theta(t)]), \tag{2}$$

where $\theta(t) < 0$ (or > 0) denotes the angle of rotation about O (or, respectively, about O'); $\theta_c = \arctan(b/h)$ is the angle shown in Fig. 1a; and $p = \sqrt{mgR/I_o}$ is a characteristic frequency parameter of the block; R is half the diagonal of the block. For a solid rectangular block the moment of inertia about its pivot point is $I_o = (4/3)mR^2$, and therefore $p = \sqrt{3g/4R}$.

In the free rocking regime the frequency of vibration depends strongly on the amplitude of rotation. Hence, the above frequency parameter p is not the eigenfrequency of the system, but merely a measure of the dynamic characteristics of the block. Table 1 summarizes the most important parameters of the problem and explains their symbols.

When a rigid body is rocking back and forth about its pivot points, it impacts on the ground and loses a part of its kinetic energy, even in a purely elastic impact. Its angular velocity right after the impact (at time t_0^+) is a fraction of that just prior to impact (at time t_0^-)

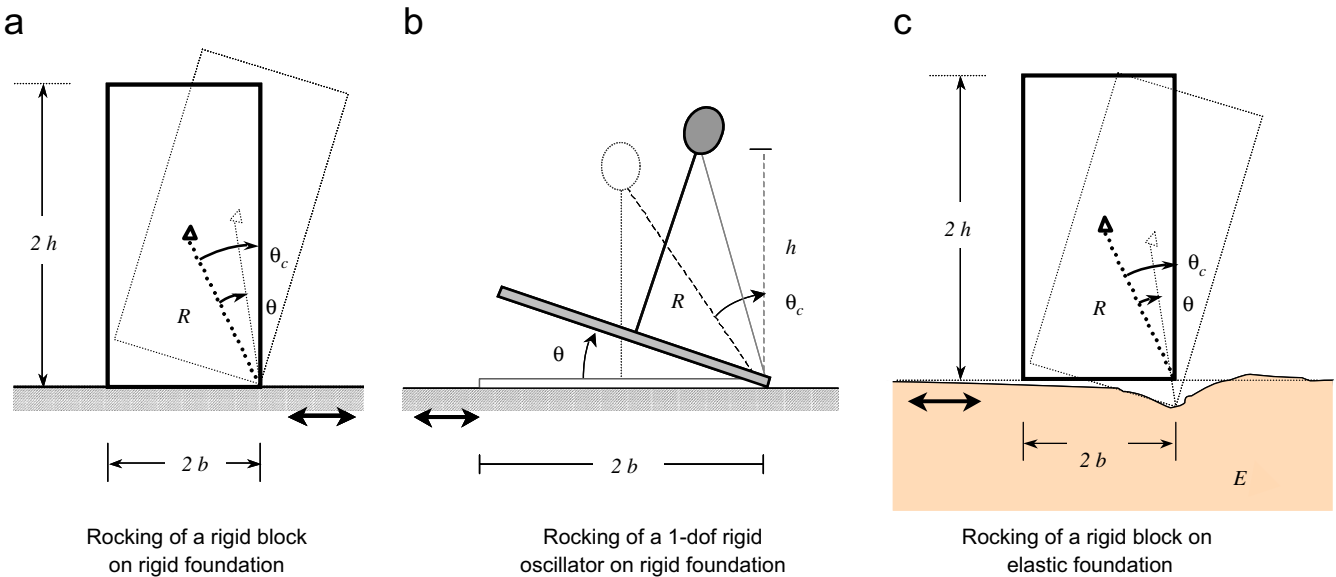


Fig. 1. The rocking systems considered in this paper. (a) Rocking of a rigid block on rigid foundation (b) Rocking of a 1-dof rigid oscillator on rigid foundation (c) Rocking of a rigid block on elastic foundation.

Table 1
Geometric characteristics and dynamic parameters considered (Nomenclature)

Parameter	Symbol
Angle of rotation	θ
Critical angle of rotation	θ_c
Aspect ratio	$b/h = \tan \theta_c$
Size parameter	$R = \sqrt{b^2 + h^2}$
Frequency parameter	$p = \sqrt{mgR/I_0}$
Period parameter	$T_p = 2\pi/p$
Ground acceleration	$A = \alpha g$
Uplifting acceleration	$A_{\text{uplift}} = \alpha_{\text{uplift}} g = (b/h)g$
Overturning acceleration	$A_{\text{over}} = \alpha_{\text{over}} g$
Excitation dominant period (frequency)	$T_E(f_E)$
Coefficient of restitution	r
Soil modulus of elasticity	E
Effective unit weight of the block	w_{eff}

$$\dot{\theta}^2(t_0^+) = r\dot{\theta}^2(t_0^-), \quad (3)$$

where r is known in the literature as *the coefficient of restitution* (often with the symbol e^2). Applying the principle of momentum preservation and neglecting energy loss during impact, we obtain for the coefficient of restitution the well-known expression [3]

$$r = \left(1 - \frac{3}{2}\sin^2\theta_c\right)^2 = e^2. \quad (4)$$

The value of the coefficient of restitution calculated by Eq. (4) is the maximum possible for a block with critical angle θ_c set on rocking motion, under elastic impact conditions. In reality, some additional energy is lost, depending on the nature of the materials at the impact surface. For a block with angle $\theta_c = 0.4$ rad (e.g., $b = 5$ m and $h = 12$ m), the maximum value of r with elastic impact is 0.60. While blocks with small aspect ratio tend to conserve most of their angular velocity, less slender blocks exhibit more geometrically “*plastic*” behavior during impact. Eventually for a value $\theta_c = 0.95$ rad ($\approx 54^\circ$) the coefficient of restitution vanishes even under purely elastic impact conditions and no rocking can be maintained after the first impact!

The governing equation of motion (Eq. (2)) along with the impact condition (Eq. (3)) can prescribe the rocking motion of a rectangular block (or any rigid structure) on a rigid base. A special case of practical interest is the rocking oscillator of Fig. 1b where the mass of the system is concentrated at point C. For a negligible rotational inertia of the mass, the moment of inertia about the rotation point O yields $I_0 = mR^2$. Hence, the frequency parameter is now

$$p = \sqrt{g/R}. \quad (5)$$

Apparently, the system of Fig. 1b, a 1-dof rocking oscillator corresponds to the rigid rectangular block, of Fig. 1a if

$$R_{1\text{dof}} = \frac{4}{3}R_{\text{block}}. \quad (6)$$

For all the analyses of a rigid rocking structure presented

next, the equation of motion was integrated numerically using the explicit algorithm and a time increment no less than 10^{-4} s.

2.1. Minimum acceleration levels for overturning under dynamic conditions

Under dynamic base excitation, exceeding the “critical” acceleration will simply initiate rocking. Whether the block will eventually overturn or not depends on its size and slenderness, as well as the kinematic characteristics and intensity of ground shaking. The major outcome of the nonlinear nature of rocking motion is that the required peak ground acceleration for overturning is a sensitive function of both the block size and the excitation frequency. This has been recognized by many researchers in the last 50 years [1–7]. Within the assumptions of small rotations and slender structures, Kirkpatrick quantified the effects of the two afore-mentioned parameters on the overturning acceleration under sinusoidal excitation, through the following simplified formula [11]:

$$a_{\text{over}} = \frac{b}{h} \sqrt{1 + \frac{\omega_E^2}{p^2}}. \quad (7)$$

Recent studies by Makris et al. [5,6] unveiled the detrimental role of long-period pulses inherent in near-fault ground motions. According to these studies a rocking block subjected to one-cycle trigonometric pulse may overturn either with one impact (*mode 1*) or without impact at all (*mode 2*), as explained in Fig. 2. A first observation is that as the frequency of excitation f_E increases, higher levels of acceleration are required to produce overturning after one impact. Much larger accelerations will lead to overturning without a single impact. The complicated nonlinear nature of the problem is also revealed from this figure: When the excitation frequency f_E is larger than a critical value f_c , the block may topple only without impact whereas for $f_E < f_c$ any one of the two overturning modes could occur. In the latter case, overturning with one impact is the critical one as it ensued always by lower levels of base acceleration. Counter-intuitively, the nonlinear nature of the problem reveals a “safe region” between the two modes, meaning that while the block overturns for a certain level of shaking, it surprisingly remains standing when the amplitude increases; for even higher levels of shaking, overturning occurs again without impact.

While cycloidal pulses are reasonable idealizations of near-fault ground motions, they cannot fully capture the effect of a slight *asymmetry* inherent to near-fault pulses. Ricker wavelets (time-histories and response spectra are presented in Fig. 3) have a distinct advantage in this respect. Thus, such wavelets are employed here to excite the rectangular block of Fig. 2, and to bring it to rocking oscillations under elastic impact conditions ($r = 0.89$). As seen in the overturning spectra plotted in Fig. 4 more

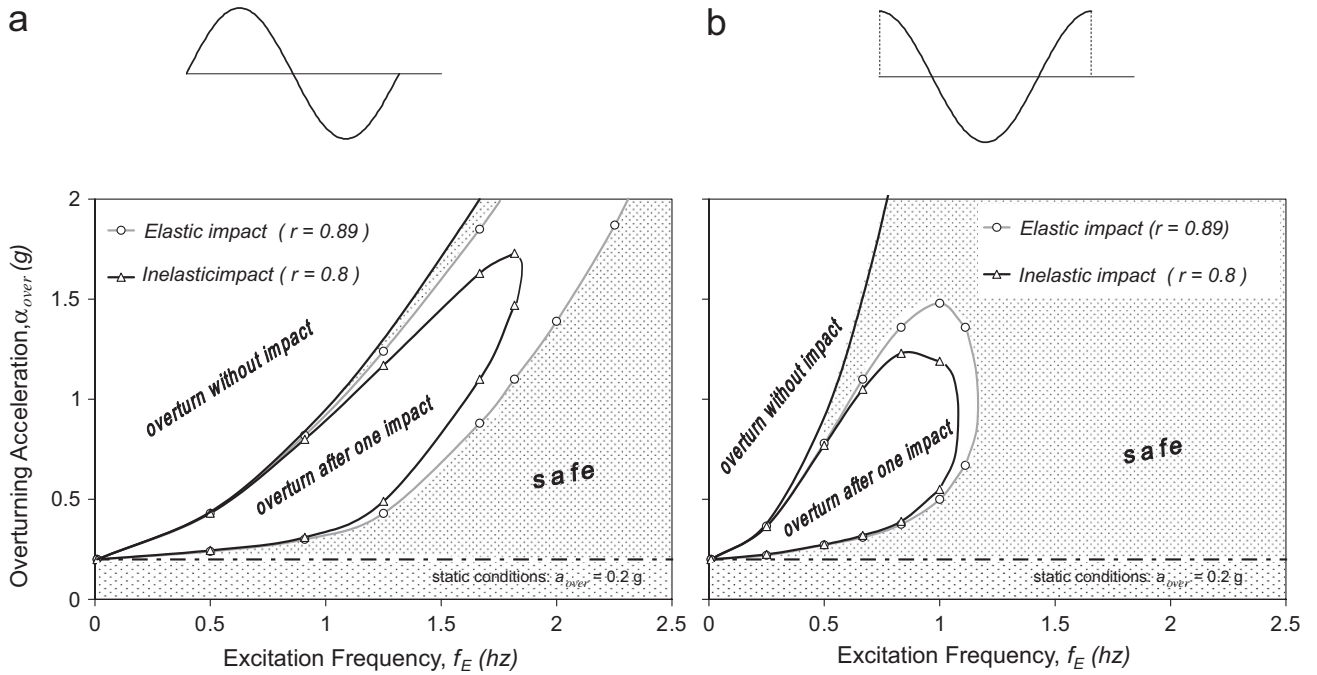


Fig. 2. Overturning spectra of a rectangular block with $2b = 1$ m, and $2h = 5$ m subjected to (a) a one-cycle-sinus and (b) one-cycle-cosinus excitation. The coefficient of restitution is 0.89 (elastic impact) and 0.8 (slightly inelastic impact). The line delineating the “overturn without impact” region from the safe region does not depend on the coefficient of restitution and thus has no circle or triangle indication.

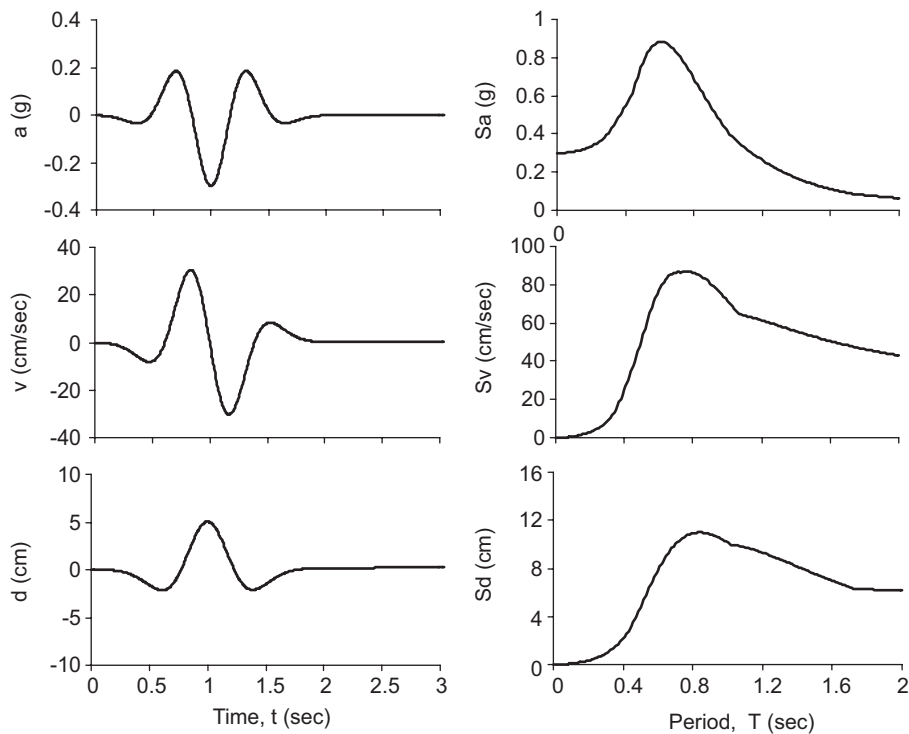


Fig. 3. Time-histories and response spectra of a Ricker-wavelet having $PGA = 0.3$ g and predominant frequency $f_E = 1.3$ Hz.

failure loops “appear” in this case. Also, there is no distinction between overturning with one or without impact as derived from the time-histories of Fig. 5. For long-duration motions ($f_E < 0.6$ Hz) the overturning accel-

eration levels between the Ricker and the cosine pulse are almost identical.

An important question following the foregoing discussion is whether high-rise buildings and tall bridge piers may

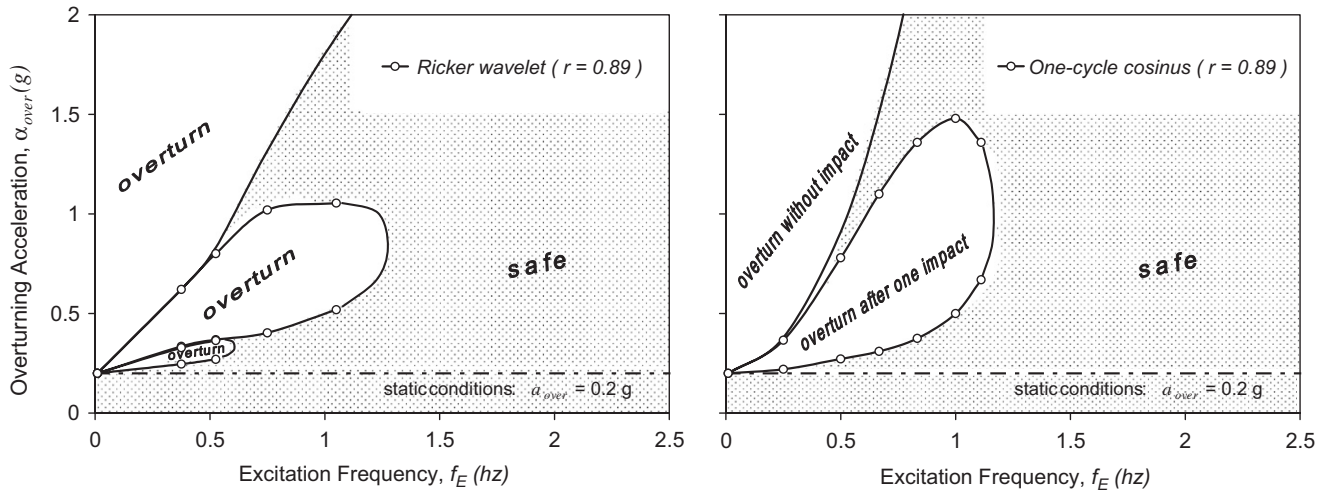


Fig. 4. Overturning spectra of a rectangular block with $2b = 1$ m and $2h = 5$ m subjected to a *Ricker-wavelet* excitation. The coefficient of restitution is 0.89 (elastic impact). Note some similarity in the shape of the various curves with the one-cycle cosine curves of Fig. 2.

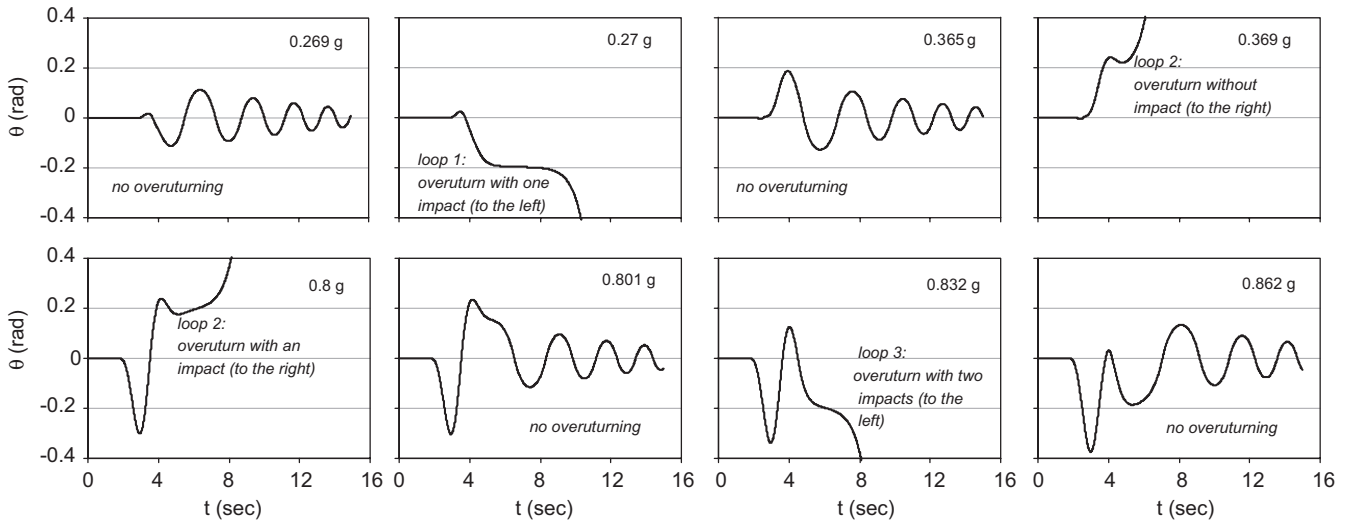


Fig. 5. Time-histories of the rocking response for a rectangular block with $2b = 1$ m and $2h = 5$ m subjected to a *Ricker-wavelet* excitation of $f_E = 0.53$ Hz. The coefficient of restitution is 0.89 (elastic impact).

safely uplift from their foundation under very strong shaking. The beneficial effect of the block size to overturning response has long been known [3,4]. This effect can be illustrated with the overturning spectrum of a rigid rocking block under a specific ground motion in terms of the frequency parameter p . As portrayed in Fig. 6 for one-cycle sinusoidal pulses with periods of 0.40 and 0.80 s, the size of the structure strongly affects the minimum acceleration levels required for toppling. Thus, small structures topple more easily than larger ones of equal slenderness. Moreover, for values of the frequency parameter p lower than unity the likelihood of even a very slender block ($h/b = 5$) to overturn is negligible even under extremely strong and long-period motions!

The interplay between slenderness and size of the structure on the overturning potential is further clarified

by computing the response of a rectangular block with a constant half-width b . In the plots of Fig. 7, the height of the block is gradually increasing so that both its slenderness (h/b) and its frequency parameter (p) keep rising. Initially, a block of $b = 0.5$ m and $h = 1.0$ m is set on rocking under a long-duration one-cycle sinus pulse of $T_E = 0.8$ s; to overturn, a peak ground acceleration of 0.7 g is needed. We next increase only its half-height h by 1 m and the overturning acceleration drops rapidly down to 0.35 g—an example of detrimental influence of slenderness. However, as the half-height of the structure is further increased, the decrease of the critical acceleration diminishes and the beneficial effect of the size parameter gradually takes over. Eventually the overturning acceleration reaches a minimum (about 0.18 g) and thereafter tends to slightly increase at higher values of h . The size effect has

now overshadowed the influence of the slenderness and become the prevailing parameter on the overturning response. Hence for a sufficiently large height, the more slender a block the less vulnerable to overturning! Thus, we can explain why large slender structures survive toppling even under severe seismic shaking. In the experimental work of Huckelbridge and Clough [10] it was made clear that for a practical building, transient uplifting response would in no way imply imminent toppling.

While the overturning hazard may not be the key issue in the seismic response of slender structures (at least if stiff soils support them), it is usually addressed in engineering practice for two different reasons: (a) toppling of non-structural elements are in many cases of special interest in seismic design procedures (for example appended equip-

ment, electrical transformers and so on [6]) and (b) for nearly a century the engineering community analyzed overturning failures observed after an earthquake to obtain rough estimates of the true intensity of (unrecorded) seismic shaking. To demonstrate how difficult it is to obtain reliably such estimates, we study the toppling of cemetery tombstones in the Athens earthquake of 1999 (Fig. 8). We had hoped that back analysis of the overturning would reveal the intensity of the unknown ground motion at this location, 2 km away from the causative fault [3,12,13].

Two different earthquake records are used as the basis of our analyses:

- The accelerogram of Sepolia station, recorded in the Athens 1999 earthquake, as a typical stiff-soil record of a moderate (M_s 5.9) magnitude event, at a distance of about 9 km from the ruptured normal fault zone. The record has a PGA = 0.36 g and dominant periods in the range of 0.15–0.25 s
- The accelerogram of Düzce in the Kocaeli 1999 earthquake, which is typical of a large (M_s 7.4) magnitude event whose strike-slip rupture is directed towards the recording soil site, and stops a few kilometers before it. The strong forward-directivity effect has given the Düzce record a characteristic long duration acceleration pulse. Its PGA = 0.37 g is similar to the one of the SPLB record, but its significant periods range from about 0.40 to at least 1.50 s.

Minimum acceleration levels required to topple the tomb are computed by numerical integration of Eq. (2) after scaling up or down each record. Throughout the analysis elastic impact conditions are considered leading to a coefficient $r = 0.928$. In the case of the Sepolia-type excitation the block can sustain rocking motion without overturning until the accelerogram is increased so that it

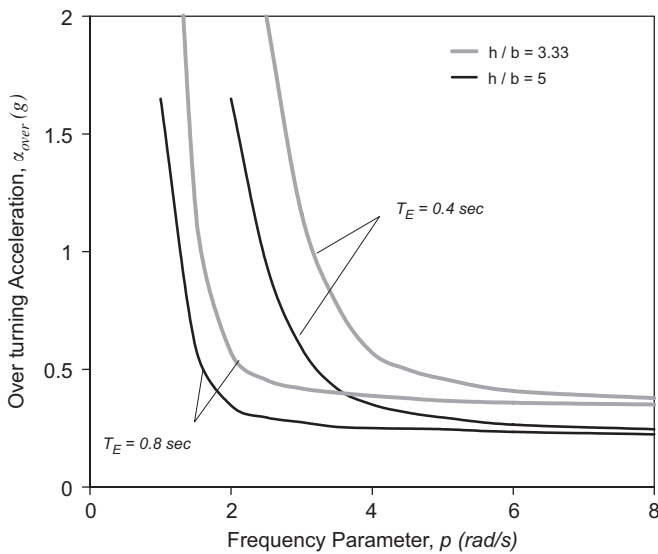


Fig. 6. Overturning spectra with respect to the frequency parameter p for blocks with aspect ratio b/h 0.2 and 0.3, subjected to a one-cycle sinus-type excitation.

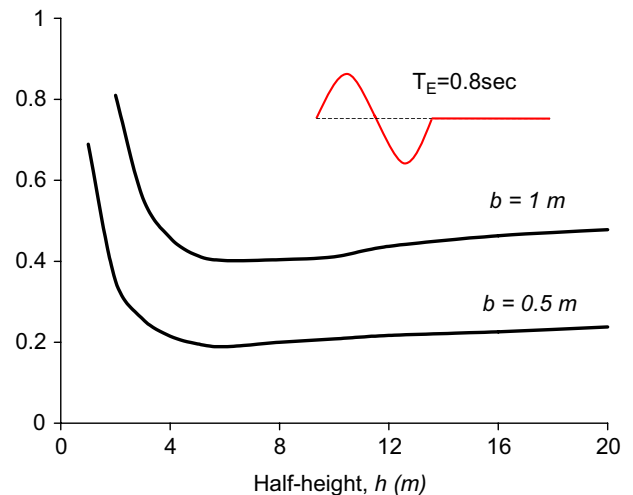
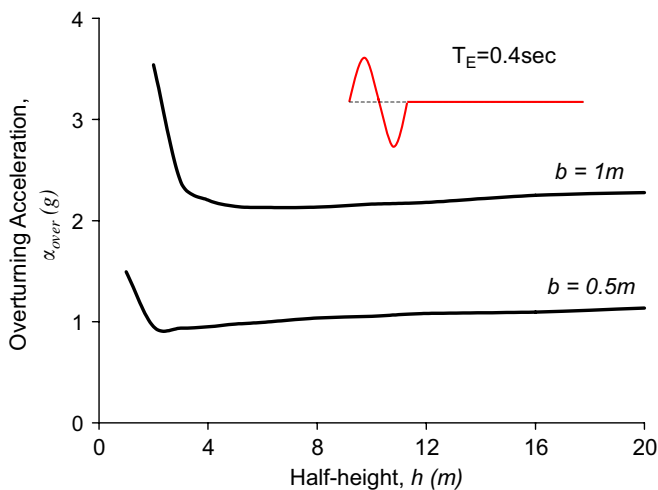
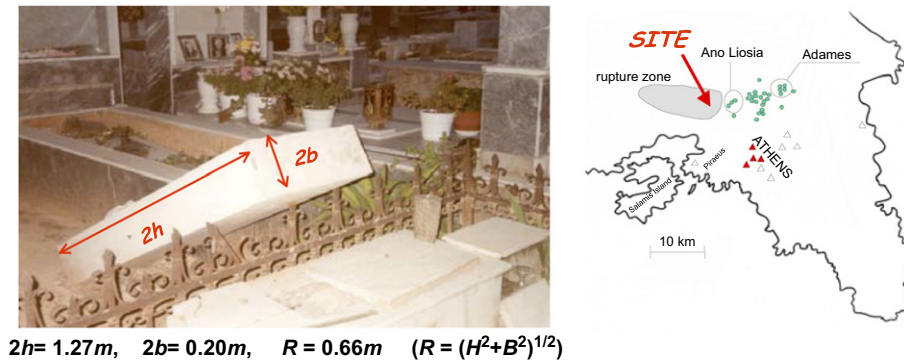


Fig. 7. Overturning spectra with respect to the half-height h for blocks with half-width 0.5 and 1 m subjected to a one-cycle sinus pulse of period 0.4 s (left) and 0.8 s (right).



$$2h = 1.27\text{ m}, \quad 2b = 0.20\text{ m}, \quad R = 0.66\text{ m} \quad (R = (H^2 + B^2)^{1/2})$$

Fig. 8. A typical free-standing tombstone that toppled after the Athens earthquake.

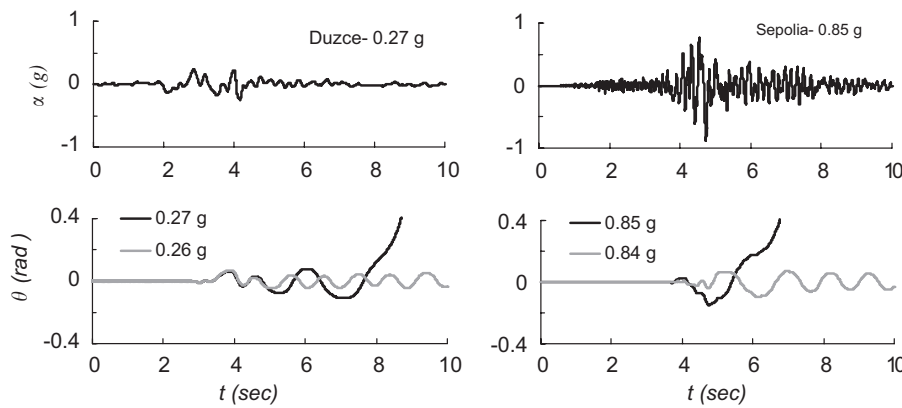


Fig. 9. The down-scaled and up-scaled accelerograms of Düzce/Kocaeli and Sepolia/Athens with the corresponding time histories of induced rotation just leading to overturning of the tombstone of Fig. 8.

acquires a PGA of 0.85 g (about 2.5 times the recorded value). By contrast, the Düzce excitation must be scaled down to a PGA of 0.27 g for overturning to occur (about 0.73 times the recorded value). Ground acceleration and rotation time histories for marginal overturning for the two records are plotted in Fig. 9. Evidently, the long-duration pulse in the Duzce record tends to reduce the overturning acceleration towards its static value ($0.20/1.27\text{ g} \cong 0.16\text{ g}$).

The rocking response of the tomb under the Sepolia-type motion is revisited next. Now the time increment of this accelerogram is artificially increased by 10% and by 20%. This leads to an increase of the predominant period of motion from 0.26 to 0.29 s and to 0.31 s, respectively. The slight modification of the excitation period has a dramatic effect on its rocking response: the overturning acceleration is reduced from 0.85 g down to 0.61 g and to 0.58 g for the two modified records! A 2-s detail of each modified time-history along with the original time-history (each one scaled to the critical acceleration) is plotted in Fig. 10.

The two distinct modes of overturning for trigonometric pulses as discussed by Makris et al. [6] are now extracted for the tombstone and plotted in the overturning spectrum of Fig. 11. For relatively low values of the excitation period T_E , a rocking block such as the tomb of Fig. 8 will not overturn even for peak ground acceleration 4 or 5 times the

pseudo-static critical acceleration (0.16 g). For values of T_E exceeding about 0.3 s the minimum PGA to overturn the block is rapidly decreasing. Eventually for sufficiently large periods ($T_E > 0.70\text{ s}$) the minimum overturning acceleration approaches the pseudo-static value. As seen in Fig. 11 the real records and the sinusoidal pulses give fairly similar results for the overturning response.

Concluding, the peak ground acceleration that toppled the cemetery block could vary from about 0.20 to 0.80 g within a period range 0.25–0.5 s. The former period is closer to the records of the Athens 1999 earthquake, which however were far-field. It is evident that the practice of estimating ground acceleration from observations of toppled and untoppled slender blocks, which has for a century been utilized to assign levels of design acceleration in many parts of the world, is meaningless in view of the strong frequency- and detail-dependence and the truly chaotic nature of rocking behavior.

2.2. Estimation of uplift

Base uplifting may have a strong influence on the performance of a structural system. Thus, the likelihood of base uplift and the estimation of the maximum rotation become of great interest. An interesting way of portraying

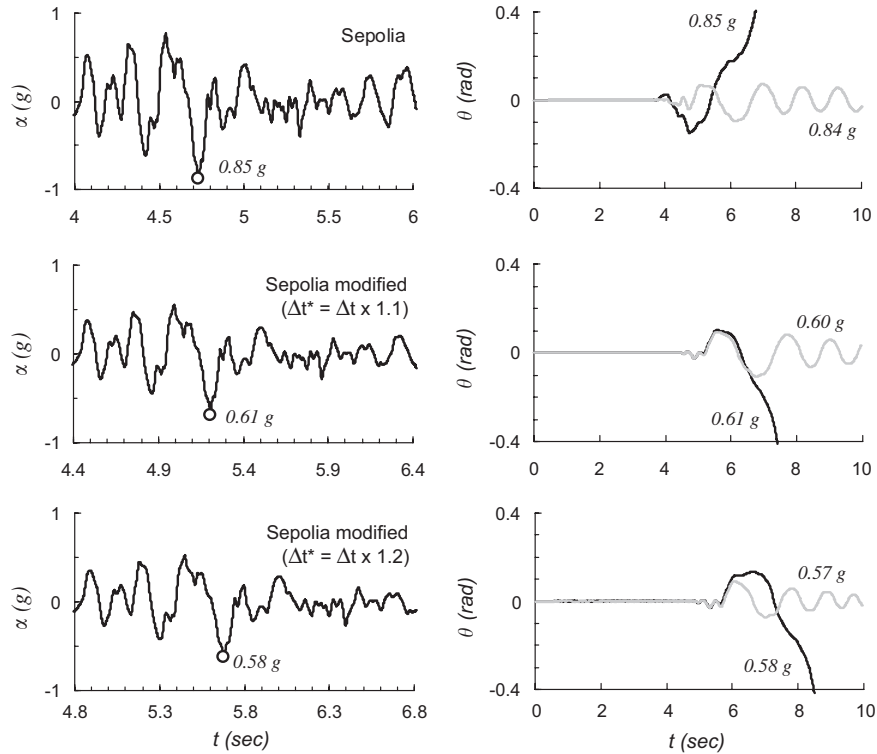


Fig. 10. Three scaled accelerograms based on the Sepolia (Athens) original record with the corresponding time histories of induced rotation just leading to overturning of the tombstone of Fig. 8. The time scale of the second and third motion has been slightly increased from that of the original record, to see the sensitivity of the toppling to the frequency content of the motion.

the response of a rigid body under rocking vibration is in the form of the *Rotation Response Spectrum* or simply “*Rocking Spectrum*”, as introduced by Makris and Konstantinidis [14]. In this, the amplitude of rotation is plotted as a function of the *period parameter* $T_p = 2\pi/p$ for a certain value of the slenderness ratio h/b or conversely of the critical angle θ_c . For a rectangular block the period parameter is $T_p = 4\pi\sqrt{R/3g} \cong 2.3\sqrt{R}$ whereas for a rigid 1-dof oscillator is $T_p = 2\pi\sqrt{R/g} \cong 2\sqrt{R}$. Note that the latter period is equal to the eigenperiod of a linearized pendulum with length R . Extending the concept, one can evaluate the rocking spectrum where the response axis is now the dynamic “eccentricity” of the vertical load on the foundation due to uplift, normalized to the half width b , of the base

$$\frac{e}{b} = 1 - \frac{\sin(\theta_c - \theta)}{\sin \theta_c} \tag{8a}$$

which, for slender blocks simplifies asymptotically to

$$\frac{e}{b} \cong \frac{\theta}{\theta_c} \tag{8b}$$

The rocking spectra for a value of critical angle $\theta_c = 0.2 \text{ rad}$ ($h/b \cong 5$) and for elastic impact (coefficient of restitution $r = 0.89$) are computed with the records of Sepolia and Düzce (Fig. 12). The far greater destructiveness of the Düzce record is evident, attributed to both its higher dominant periods and its long-duration acceleration

pulse, which is associated with large incremental velocity (in excess of 100 cm/s)—the end result of forward rupture directivity. Despite its equally large PGA, the Sepolia record is much easier for a slender block to safely undergo. The Düzce record can topple all rectangular blocks with $R < 1.2 \text{ m}$ while under the Sepolia excitation even smaller blocks (with R up to 0.2 m) would exhibit rocking motion without toppling. Moreover, the amplitude of rotation for all non-toppled blocks is larger under a Düzce-type ground motion.

The detrimental influence of a long-duration excitation pulse and the beneficial effect of the block size can also be unveiled with the use of a Ricker wavelet as excitation [15]. A comprehensive parameter study is performed to this end. The response is measured in terms of the angle of rotation $\theta (\leq \theta_c)$; the only problem parameters on which it depends are: the critical angle $\theta_c = \arctan(b/h)$, or equivalently the uplifting acceleration $A_c = a_c g$ with $a_c = b/h$; the characteristic period T_p ; and the period and peak acceleration of the Ricker excitation, T_E and $A = ag$, respectively. A unique relationship has been found between the *dimensionless parameter*

$$\Pi_\theta = \left(\frac{\theta}{\theta_c}\right) \left(\frac{T_p}{T_E}\right)^2 = \Theta \Omega^2 \tag{9}$$

and the ratio $a_c/a = A^{-1}$ which could be interpreted as the instantaneous factor of safety against uplifting. $\Theta = \theta/\theta_c$

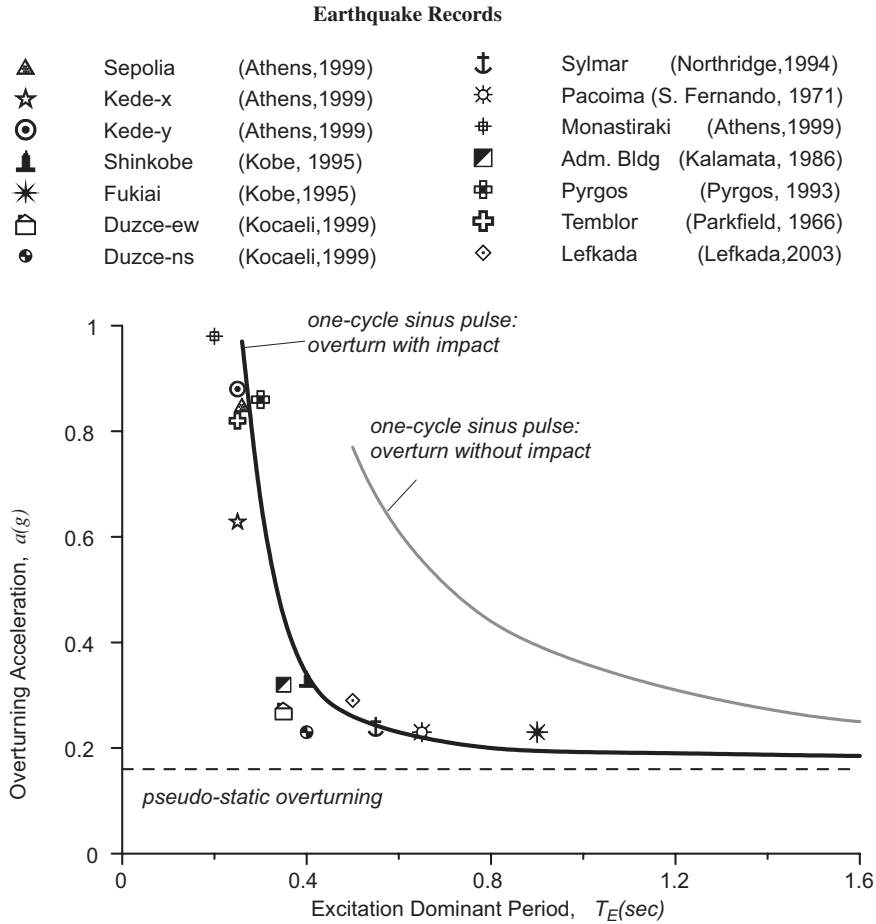


Fig. 11. Overturning spectra of the cemetery tomb ($h/b = 6.35$) for one-cycle sinus-type and for numerous ground motions used as earthquake excitation.

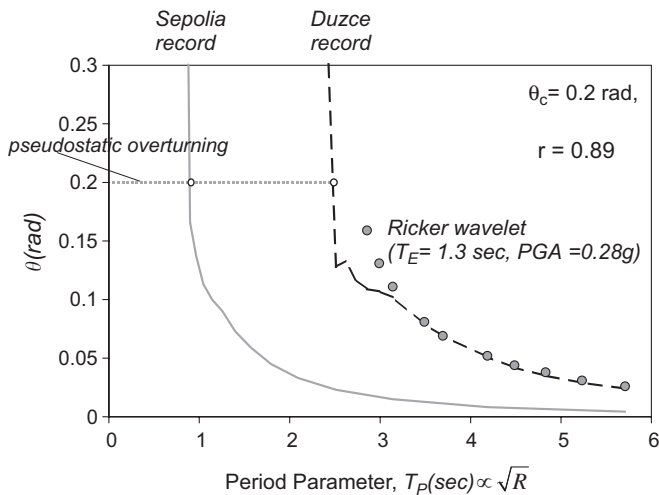


Fig. 12. Rocking spectra for blocks with $h/b = 5$ (critical angle $\theta_c = 0.2$ rad) subjected to the unscaled records of: (a) Düzce, in the Kocaeli Earthquake and (b) Sepolia, in the Athens Earthquake.

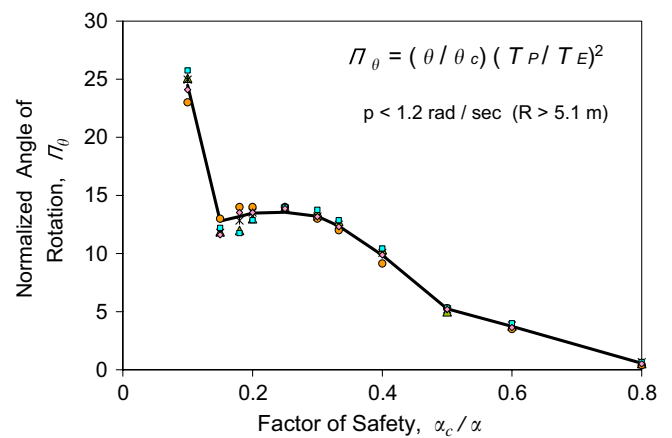


Fig. 13. Normalized angle of rotation for rigid blocks subjected to Ricker pulse-type excitation with respect to the factor of safety for uplift: α_c/α . Note that $\theta_c = \arctan \alpha_c$. (The coefficient of restitution for all analyses is $r = 0.80$.)

and $\Omega = (T_p/T_E) = (\omega_E/p)$ are, respectively, the dimensionless uplifting rotation and frequency. Fig. 13 plots this unique relationship $\Pi_\theta = \Pi_\theta(\alpha_c/\alpha)$.

The rocking amplitude developed under a Ricker-type excitation is compared next with two typical near-fault

earthquake accelerograms: (a) the aforementioned Düzce record and (b) the Pacoima dam record from the San Fernando earthquake (1971). The rocking spectra are plotted in Fig. 14 for four values of the frequency parameter p . Evidently the use of low-frequency Ricker

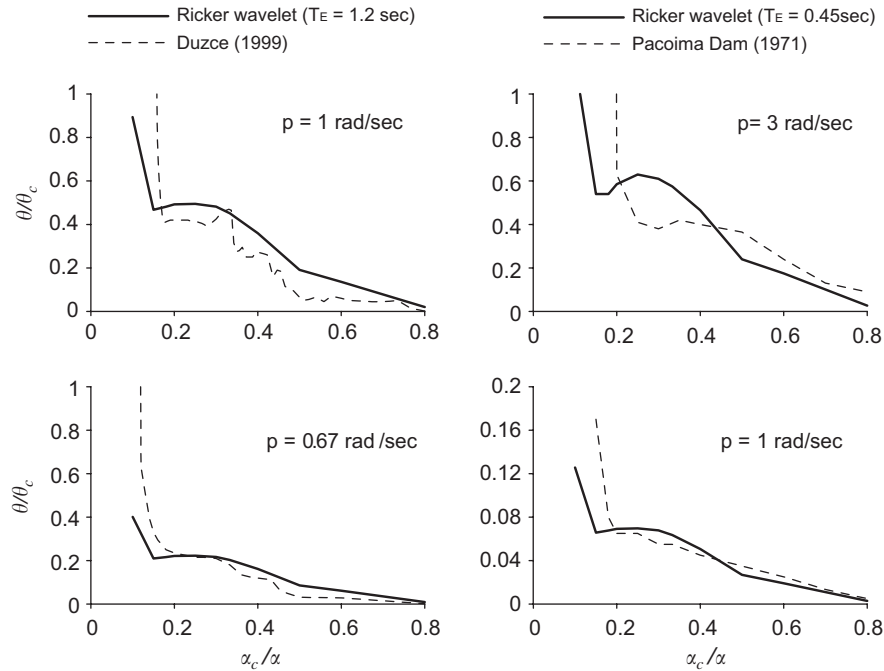


Fig. 14. Comparison of rocking spectra for blocks subjected to Ricker wavelets and two near-fault earthquake records. The coefficient of restitution is $r = 0.80$.

wavelets in representing near-fault ground motions is justified.

For design purposes the peak rotational angle of a structure subjected to near-fault ground motion can be estimated conservatively with the following expressions: for $a_c/a > 0.3$

$$\Pi_\theta \cong 20 \left(1 - \frac{a_c}{a}\right) \quad (10a)$$

from which

$$\frac{\theta}{\theta_c} \cong 20 \left(1 - \frac{a_c}{a}\right) \left(\frac{T_E}{T_P}\right)^2 \quad (10b)$$

for $0.15 < a_c/a < 0.3$

$$\Pi_\theta \cong 14 \quad (10c)$$

from which

$$\frac{\theta}{\theta_c} \cong 14 \left(\frac{T_E}{T_P}\right)^2 \quad (10d)$$

(Note that these two equations apply only to sufficiently large structures: $p < 1.2$ rad/s). It is obvious that for a given slenderness ratio, the response ratio θ/θ_c increases with the square of the dominant excitation period but decreases in proportion to the size R of the block.

3. Uplifting and overturning on elastic soil

Consider now the system of Fig. 1(c): a rigid block supported on an elastic homogeneous half space of Young's modulus E , Poisson's ratio ν , and damping ratio ξ , subjected to horizontal base excitation $A(t) = a(t)g$. Due

to soil compliance, the block can now undergo rotational motion without uplift (so long as rotational amplitudes remain below the critical angle). For large amplitudes, the rocking response alternates between the modes of full-contact and uplift. The critical angle for uplift is given by the following expression:

$$\theta_{\text{uplift}} = \frac{M_{\text{uplift}}}{K_R}, \quad (11)$$

where the uplifting moment M_{uplift} is a fraction of the moment capacity $M_{\text{ult}} (= Nb)$ of the foundation–soil system

$$M_{\text{uplift}} = \eta M_{\text{ult}}. \quad (12)$$

Note that for a rigid beam on Winkler foundation the uplifting moment is $M_{\text{uplift}} = Nb/3$. Therefore, it is $\eta = \frac{1}{3}$ from Eq. (12). However, with an elastic continuum for representing the soil and plane strain conditions of loading this coefficient increases up to 0.5 [16]. In this case and recalling [17,18] the expression for rocking stiffness of a strip foundation on a homogeneous half-space ($K_R = \pi G b^2 / 2(1 - \nu)$), the rotation at incipient uplifting becomes

$$\theta_{\text{uplift}} = \frac{Nb}{2K_R} = \frac{4w_{\text{eff}}h(1 - \nu)}{\pi G} = \frac{8w_{\text{eff}}h(1 - \nu^2)}{\pi E}. \quad (13)$$

Due to soil flexibility the following parameters should be taken into account:

- the soil properties E , ν , ρ ,
- the effective unit weight of the block: $w_{\text{eff}} = N/4bh$, where N is the block's weight (per unit length),
- the presence of bedrock at a shallow depth.

In the present study the dynamic analysis of the rocking response is implemented with a finite element discretization using *Abaqus* [19]. The structure and the underlying soil are represented with plane-strain elements. An advanced contact algorithm has been adopted to incorporate potential slipping or uplifting of the foundation, considering purely elastic impact. Wherever the supporting soil is treated as a homogeneous halfspace, two-dimensional infinite elements are applied to model the boundary conditions.

The compliance of the supporting soil introduces additional modes of deformation. The structure can now rotate without necessarily uplifting (the linear component of the motion). In addition, uplifting (the nonlinear component of rocking) may also take place. What is more, in soft soil the impact during rocking is more “absorbing” as radiation and hysteretic damping are generated in the soil. Thus attenuation of the motion is faster. Fig. 15 illustrates the two components of rocking for a slender $h/b = 5$ block supported on three different elastic soils, having $E = 100$ MPa (very stiff), 20 MPa (moderately stiff), and 5 MPa (very soft). Two values of the a_c/a ratio, 0.20 and 0.50 are considered; since $a_c = 0.20g$ the implied Ricker peak accelerations a are, respectively, 1.0 and 0.4 g. The gray curves stand for the time history of total rotation, defined as $\Delta_{total}/2b$, while the black curves are for the rotation component due to uplifting, defined as $\Delta_{uplift}/2b$. $\Delta_{total} = \Delta_{uplift} + \Delta_{elastic}$ is the vertical distance between the

two corners of the foundation (O and O’). It is seen that in the very stiff, nearly undeformable soil ($E = 100$ MPa), rotation is due almost exclusively to uplifting: the two curves almost coincide. On the other end of the spectrum, on soft soil ($E = 5$ MPa) uplifting contributes only part of the total rotation of the block, and essentially only during the strong excitation pulse. The ensuing free oscillations are simply the rotational vibrations of the block on a homogeneous elastic layer, at periods ($\cong 2$ s) well above the cut-off period of the soil stratum: hence neither rotation nor hysteretic damping are present, and the free vibrations continue unattenuated [18]. Note also in Fig. 15 that for $a_c/a = 0.2$ the peak uplifting angle (nonlinear component) is 0.08 rad in case of $E = 100$ MPa, while for $E = 20$ and 5 MPa the uplifting response decreases to 0.06 and 0.04 rad, respectively. On the contrary, the linear component of the motion (due to soil compliance) becomes larger as soil compliance increases, compensating to a larger extent for the reduced uplifting.

To investigate the effect of soil compliance on rocking response, the peak angle of rotation is plotted in Fig. 16 for a range of E -values (5–1000 MPa) and three different block sizes ($R = 2.8, 3.5,$ and 5.1 m). For very high values of the modulus of elasticity, the amplitudes of rotation converge to the limiting case of the amplitude on rigid base ($\theta_{rigid} \cong 0.03$ rad, $\cong 0.05$ rad, and $\cong 0.08$ rad for each of the aforementioned three R -values, respectively). Upon decreasing E , the effect of soil deformability leads under-

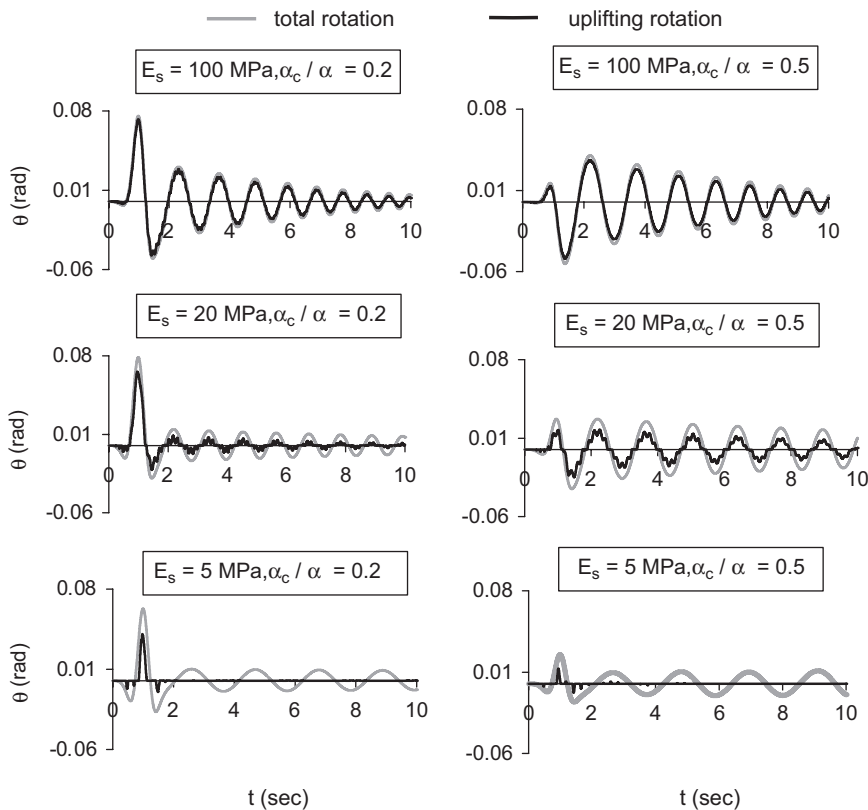


Fig. 15. Uplifting behavior of a block with $b = 1$ m and $h = 5$ m induced by a Ricker pulse of frequency 1 Hz. Gray curves show the overall response; black curves the uplifting part of the response. Plane-strain conditions prevail.

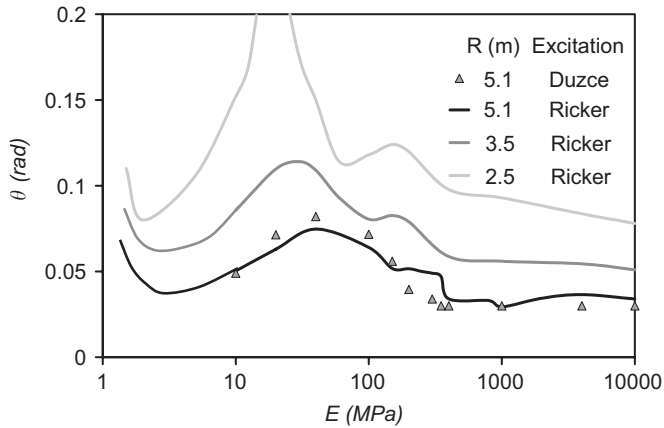


Fig. 16. Amplitudes of rotation for three rectangular block-type structures with constant aspect ratio ($h/b = 5$ or $\theta_c = 0.2$ rad). The excitation is a Ricker wavelet (PGA = 0.3 g, $T_E = 1.3$ s) and the Duzce record (PGA = 0.37 g).

standingly to greater values of the maximum angle, which can go up more than 2 times the rigid base value. For even smaller values of E , less than about 10–15 MPa, the increased softening of the soil is beneficial, leading to smaller θ -values! In all these cases ($E > 5$ MPa), the structure oscillates in rocking without overturning, despite the pseudo-statically predicted toppling. However, for very small values of E , less than about 2–5 MPa, the trend changes again and θ increases with increasing E . Failure is now possible since the large deformability of the soil leads to significant rotation that triggers deleterious P - δ effects. A quite interesting rocking behavior is revealed when smaller structures of equal slenderness are considered as also shown in Fig. 16. In this way two smaller blocks are considered with base widths 1.4 m and 1.0 m, and heights 7.0 m and 5.0 m, respectively; therefore the critical angle of rotation *remains constant*. Only the dimensions of each block, described through the half-diagonal $R = (b^2 + h^2)^{1/2}$, change (from 2.5 to 5.1 m). The following trends are worthy of note in this figure: (a) the overall size of the block affects strongly its rotation; the smallest of the three blocks undergoes the largest rotation for all values of E and it in fact overturns for $E \cong 15$ MPa; and (b) the variation of θ_{max} with respect to the soil modulus is *not* monotonic; it exhibits a peak at $E \cong 15$ –30 MPa where the rocking period $T_R = 2\pi\sqrt{J_b/K_R}$ is tuned to the excitation period (1.3 s), and tends again to become very large as E tends to zero. A secondary peak is also noticed at $E \cong 150$ –200 MPa. Nevertheless, the maximum rocking angle in case of soft soil would in most cases be not more than 1.5–2 times the corresponding “rigid-base” value.

Finally, Fig. 17 shows the dependence of the peak angle of rotation, θ on the ratio a_c/a , for combinations of two values of soil Young’s modulus, $E = 20$ and 5 MPa, and two values of the effective unit weight of the block, $w_{eff} = \gamma_w$ and $\gamma_w/4$, where $\gamma_w =$ the unit weight of water. The latter value of w_{eff} is typical for the unit weight of a building, while the former value represents a heavier

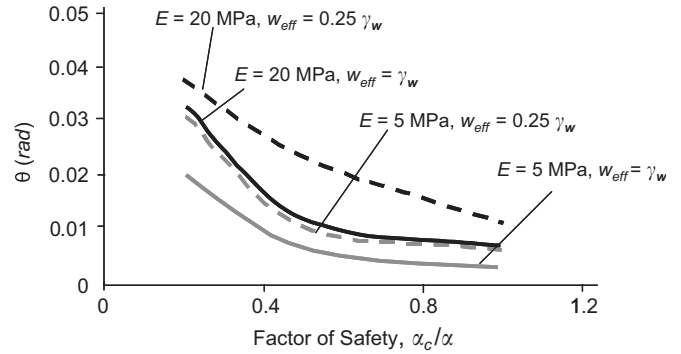


Fig. 17. Influence of the soil stiffness and the unit block weight on the rocking spectra for a block with $b = 1$ m and $h = 5$ m. The excitation is a Ricker pulse of characteristic frequency 1 Hz.

structure, such as a water tank. The following trends are worthy of note:

- The values of θ do *not* vanish for $a_c/a = 1$ due to “elastic” rotation of the foundation.
- The softer soil leads to small overall rotation, since in this particular case the natural rotational frequency of the block on elastic soil is much smaller than the dominant frequencies of the Ricker excitation. (With the stiffer soil, the corresponding frequencies are closer to the excitation frequencies.)
- Increasing the unit weight w_{eff} of the structure also reduces the peak rotation, since the block’s natural rotational frequency further decreases. In fact, in this case θ is roughly proportional to E and inversely proportional to w_{eff} .

This figure was only meant to be an example of the interplay among the various soil and structure parameters. The results should not be unduly generalized.

4. Conclusions

The paper investigates the rocking and overturning response of slender rigid structures allowed to uplift. The following concluding remarks can be drawn:

- (1) Under static conditions only the *slenderness* of a structure is the decisive parameter for toppling. In dynamic terms however, the *size* and the *slenderness* of the structure, as well as the nature of base shaking affect the overturning potential. For large structures, size effects are prevailing in such a way that a large slender block can safely undergo a certain excitation while a less slender but smaller block overturns. This explains why large structures survive toppling even under very strong seismic shaking, far greater than the pseudostatically required for overturning. Moreover, it is not only the dominant frequency but also the nature and especially the asymmetry of a base excitation that have a strong effect on the overturning potential.

Therefore, the practice of estimating ground shaking levels by analyzing observations of toppled and untoppled slender blocks after an earthquake is meaningless.

- (2) For relatively large structures on rigid foundation, seismic uplifting response can be reliably predicted. In such a case the rocking amplitude can be normalized and estimated through suitable dimensionless charts. An example is given in this study for pulse-type motions idealized as Ricker wavelets. The seismic response of smaller structures however, is rather “chaotic” and can hardly be predicted with confidence if the details of the base motion are not known with accuracy.
- (3) For an elastic supporting soil, there is no definitive relation between rocking response and the a_c/a ratio. This is especially true with soft soils when the linear component of the response (full-contact regime due to soil compliance) becomes important. This is one of the consequences of the strong geometric nonlinearity of the problem. The additional complication arising from nonlinear material behavior of soils and the mobilization of bearing capacity mechanisms is beyond the scope of this paper, but it has been introduced in Refs. [20,21].

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