PREDICTION OF OBSERVED BRIDGE RESPONSE WITH SOIL-PILE—STRUCTURE INTERACTION

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ABSTRACT: A simple integrated procedure to analyze the problem of soil–pile–foundation–superstructure interaction is presented. The procedure combines the available theories for the computation of the dynamic impedances and kinematic–seismic response factors of pile foundations with a simple six-degree-of-freedom structural model. The procedure is used to predict the response of the Painter Street Bridge located at Rio Dell, California, which was excited by the 1992 Petrolia earthquake. The predicted response with the proposed procedure is in very good agreement with recorded data, and the significance of considering the frequency dependent pile-foundation impedances in predicting the superstructure response is demonstrated.

INTRODUCTION

In the last two decades, several numerical and analytical methods have been developed to compute the dynamic stiffnesses and the seismic response of pile foundations accounting for pile-soil-pile interaction. This pile-to-pile interaction is frequency dependent, resulting from waves that are emitted from the periphery of each pile and propagate to “strike” the neighboring piles. Today, numerous rigorous and approximate procedures are available to compute the response of piles and pile groups under dynamic loads. A recent comprehensive review on the subject has been presented by Novak (1991). On the other hand, during the same period, the software for structural analysis has been considerably developed and expanded so that complex structures with sophisticated behavior can be rigorously analyzed.

Despite these advances, very little is known about the significance of pile-foundation—superstructure interaction, even for important structures like bridges. For instance, in many cases the sweeping assumption that the pile group is a fixed, monolithic support prevails. Possible reasons for the lack of established procedures in analyzing structures supported on pile foundations is that most of the research results published on the dynamic response of pile foundations is scattered and presented in such a form that is of little use to the structural engineer; and some of the most reliable methods to analyze the response of pile foundations rely on proprietary computer codes.

The aim of this paper is twofold: To present an integrated procedure to analyze the problem of soil–pile-foundation–superstructure interaction using the existing available theories for the computation of the dynamic impedances...
ances and kinematic-seismic-response factors; and to investigate the significance of considering the frequency dependence of pile-foundation impedances to the response of the superstructure. The presented methodology is used to predict the response of an existing bridge, the Painter Street Bridge in Rio Dell, California, which has been instrumented since 1977 by the California Division of Mines and Geology and survived the 1992 Petrolia earthquake. The predicted response using the proposed procedure is in very good agreement with recorded data, and considerable supporting evidence on the significance of the soil–pile-foundation–superstructure interaction is provided.

**PAINTER STREET BRIDGE**

The Painter Street Bridge, located near Rio Dell in Northern California, is a continuous, two-span, cast-in-place, prestressed post-tensioned-concrete, box-girdered bridge. It is a typical concrete bridge constructed in 1973 and spans a four-lane highway. The structure has one span of 146 ft (44.5 m) and one of 119 ft (36.3 m). It is 52 ft (15.8 m) wide. The two end abutments and the two-column bent are skewed at 39°. The columns are approximately 20 ft (6.0 m) high. The bent is supported by two pile groups, each consisting of 20 (4 × 5) driven concrete friction piles. An elevation and plan view of the Painter Street Bridge, together with the location of the accelerometers, is presented in Fig. 1. The cross-section of the bridge and a plan-view of the pile-group is presented in Fig. 2.

The Painter Street Bridge was instrumented in 1977 by the California Division of Mines and Geology. Several earthquakes from 1980 to 1987 ranging in magnitude from 4.4\(M_s\) to 6.9\(M_s\) have produced significant accelerograms, the peak values of which are summarized by Maroney et al. (1990).

Maroney et al. (1990) utilized these records in conjunction with a number of finite-element and lumped-parameter (stick) models of the entire bridge. However, none of these models accounted for soil-foundation–superstructure interaction. At each abutment, soil-wall interaction was modeled through a single real-valued transverse spring, the stiffness of which was backfigured from the interpreted fundamental natural period, \(T = 0.30\) s, in lateral vibration.

On April 25, 1992, the bridge was severely shaken by the Petrolia Earthquake (magnitude \(M_s = 7.1\), distance to fault \(R = 18\) km). Motions were recorded in all accelerographs, including the one at the free field (channels 12–14), atop the footing of one pier (channels 1–3), and at the underside of the bridge girder directly above the pier (channel 7). The location of all accelerographs is depicted in Fig. 1, together with the peak recorded acceleration (PRA) for each channel. Of particular interest in this paper are the records of channels 3 (PRA = 0.48 g) and 7 (PRA = 0.92 g). Despite the very high levels of accelerations, the bridge suffered only minor damage. The N-S (channel 14), E-W (channel 12) and vertical (channel 13) free-field records of the Petrolia Earthquake are shown in Fig. 3.

An interesting study of the response of the Painter Street Bridge to the Petrolia earthquake was presented recently by Sweet (1993), who developed a finite-element model encompassing the whole bridge and a large volume of surrounding and supporting soil. Inelastic soil behavior was modeled using an incremental plasticity model developed by the writer. But all this sophistication was inconsistent with the modeling of the pile group behavior. Indeed, such modeling was based on the sweeping assumption that no pile-
soil-pile interaction occurs at the 3-ft spacing; the 20-pile group behavior was assumed to be merely 20 times that of a single pile. In fact, pile-to-pile interaction is expected to play a very substantial role in pile group response, given the very close relative spacing of the piles \( (S/d = 2.60) \).

The philosophy behind the model developed in this paper is diametrically opposite to that of Sweet (1993). The simplest possible model of the pier-deck system is studied by representing the pile-group foundation with a set of "springs" and "dashpots." However, the frequency-dependent values of
these springs and dashpots are computed in a rational, physically motivated manner, accounting fully for pile-to-pile interaction.

The bridge is ideal, as depicted in Fig. 4, with a plane six-degree-of-freedom (DOF) lumped-parameter model. The values $m_s, m_f, I_s,$ and $I_f$ are the masses and moments of inertia of the superstructure and the foundation. In the case of a multispans bridge, $m_s$ and $I_s$ are the mass and moment of inertia of the deck corresponding to one span plus some contribution from the bridge-pier. Values $K_X^s, K_{xz}^s, K_R^s,$ and $K_{xR}^s$ are the horizontal, vertical, rotational and cross horizontal-rotational stiffnesses of the superstructure. For the section of the superstructure considered, $K_X, K_Z, K_R$ and $K_{XR}$ are the combination of the stiffnesses of the bridge pier and the bridge deck. This is a reasonable approximation in the case where the abutments of the
FIG. 4. Pile-Foundation—Superstructure Idealization
bridge and the foundation of the bridge pier experience the same input motion. For relatively short bridges this is indeed a good approximation and the model is realistic. For long bridges, the abutment motion might be incoherent and a more sophisticated model would be more appropriate. However, for bridges with very long spans the contribution from the deck stiffness might be negligible and the model becomes again realistic, since the entire horizontal stiffness of the superstructure is essentially due to the bridge pier only. Values \( C_\text{f} \) and \( C_\text{y} \) are the damping constants of the superstructure, and are assumed to be linear viscous. Values \( K_\text{f} \), \( K_\text{y} \), \( K_\text{z} \), and \( K_\text{y} \) are the horizontal, vertical, rocking and cross-horizontal-rocking impedances of the pile foundation and are complex valued quantities. The presented procedure is not restricted to the preceding simple structural model described. Additional degrees of freedom for the structure could be incorporated to account for the motion of end abutments and the torsional vibration of the bridge about the vertical axis.

SOIL–PILE–FOUNDATION–SUPERSTRUCTURE ANALYSIS

Assuming linear soil-foundation–superstructure response, the system analysis under seismic excitation can be performed in three consecutive steps: (1) Obtain the motion of the foundation in the absence of the superstructure—the so-called foundation input motion, which includes translational as well as rotational components; (2) determine the dynamic impedances (springs and dashpots) associated with swaying, vertical, rocking and cross-swaying-rocking oscillations of the foundation; (3) compute the seismic response of the superstructure supported on the springs and dashpots of step 2 and subjected at ground base to the foundation input motion of step 1. For the first two steps, several alternative formulations have been developed and published in the literature including finite-element (Blaney et al. 1976; Kagawa and Kraft 1982; Gazetas 1984), boundary-element (Banerjee and Sen 1987; Fan et al. 1991; Kaynia 1982; Kaynia and Novak 1992; Waas and Hartman 1984), and semianalytical-type methods (Tajimi 1977; Kobori et al. 1991; Nogami 1983), and a number of simplified analytical solutions (Dobry and Gazetas 1988; Makris and Gazetas 1992; 1993), among others. Herein, use is made of the simplified, yet reliable analytical method developed in the last three references.

Foundation Input Motion: Step 1

The input motion to the system is induced at the foundation. In general, the support motion induced to the foundation is different than the free-field seismic motion. This difference is due to the scattered wave field, which is generated between soil- and pile displacement. Nevertheless, for motions that are not rich in high frequencies, the scattered field is weak, and the support motion can be approximately considered equal with that of the free field (Fan et al. 1991).

In the case of the Painter Street Bridge, the soil deposit has an average shear velocity \( V_s \approx 200 \text{ m/s} \) (Heuze and Swift 1991) and the pile diameter \( d = 0.36 \text{ m} \). Accordingly, even for the high-frequency content of the input motion \( f \approx 10 \text{ Hz} \), the dimensionless frequency \( a_0 = 2\pi fd/V_s \) is only of the order of 0.1; and therefore, the kinematic-seismic-response factors (headgroup displacement over free-field displacement) in all vibration modes are very close to one (Gazetas et al. 1992). Based on the foregoing supporting evidence, the excitation input motion on the pile foundation of the bridge has been assumed equal to that of the free-field motion.
Dynamic Impedances of Pile Foundations: Step 2

The dynamic stiffness of a pile group, in any vibration mode, can be computed using the dynamic stiffness of the single pile in conjunction with dynamic interaction factors. This method, originally introduced for static loads by Poulos (1968), and later validated for dynamic loads by Kaynia and Kausel (1982), Sanchez-Salinerio (1983), and Roesset (1984), can be used with confidence—at least for groups not having a large number of piles (say less than 50). Dynamic interaction factors for various modes of loading are available in the form of ready-to-use, nondimensional graphs (Gazetas et al. 1991), while in some cases closed-form expressions are derived from a beam on Winkler foundation model in conjunction with simplified wave propagation theory (Makris and Gazetas 1992). As an example, the horizontal and vertical dynamic interaction factor for two fixed-head piles in a homogeneous stratum takes the form

\[
\alpha_x(S, \theta) = \frac{3}{4} \psi(S, \theta) \frac{k_x(\omega) + \imath \omega c_x(\omega)}{k_x(\omega) + \imath \omega c_x(\omega) - \imath m \omega^2}
\]  

(1)

\[
\alpha_z(S) = \left( \frac{d}{2S} \right)^{1/2} \exp \left[ -\frac{\beta_s}{V_s} \left( S - \frac{d}{2} \right) \right]
\]

(2)

where \( k_x \) and \( c_x \) are distributed spring and dashpot coefficients; and \( \psi(r, \theta) \) is an approximate attenuation function proposed in the above mentioned reference. Furthermore, \( i = \sqrt{-1} \); \( S \) is axis-to-axis distance between two piles; \( \theta \) is angle between the direction of loading and the line connecting the axis of the two piles; \( \beta_s \) is hysteretic damping in the soil; and \( V_s \) is shear wave velocity of the soil.

Knowledge of the dynamic stiffnesses of the single pile and the dynamic interaction factors between any two piles allows the computation of the dynamic stiffnesses of a group of piles using the concept of superposition. Let \( y_i \) be the horizontal displacement of pile \( i \) belonging to a group of \( N \) piles. Superposition of displacements leads to

\[
y_i = \sum_{j=1}^{N} \alpha_{x(i,j)} y_j
\]

(3)

where \( \alpha_{x(i,j)} \) is given from (1). Of course if \( j = i \), then \( \alpha_{x(i,i)} = 1 \). The value \( y_j = \) displacement of a single (solitary) pile; it is obtained as

\[
y_i = \frac{F_i}{K_X^{[1]}}
\]

(4)

where \( F_i \) is force that this pile carries; and \( K_X^{[1]} \) is horizontal dynamic stiffness of the single fixed-head pile. Since all piles are connected with a rigid cap, the displacement group, \( y^{[G]} \), is equal to \( y_i \), for all \( i \). Substitution of (4) into (3) gives

\[
y^{[G]} K_X^{[1]} = \sum_{j=1}^{N} \alpha_{x(i,j)} F_j
\]

(5)

Repeating (5) for all \( N \) piles of the pile group, one obtains the matrix equation
\[
\begin{bmatrix}
\alpha_{X(11)} & \alpha_{X(12)} & \cdots & \alpha_{X(1N)} \\
\alpha_{X(21)} & \alpha_{X(22)} & \cdots & \alpha_{X(2N)} \\
\vdots & \vdots & \ddots & \vdots \\
\alpha_{X(N1)} & \alpha_{X(N2)} & \cdots & \alpha_{X(NN)}
\end{bmatrix}
\begin{bmatrix}
F_1 \\
F_2 \\
\vdots \\
F_N
\end{bmatrix} = y^{G|} K_{X}^{[G]}
\begin{bmatrix}
1 \\
1 \\
\vdots \\
1
\end{bmatrix}
\] (6)

Eq. (6) can be solved for the force vector

\[
F_i = y^{G|} K_{X}^{[G]} \sum_{j=1}^{N} \varepsilon_{X(i,j)}
\] (7)

where \(\varepsilon_{X(i,j)}\) = element of the inverse of matrix \(\alpha_{X(i,j)}\). Since \(P_i^{G} = K_{X}^{[G]}\)
\(y^{G} = \sum_{i=1}^{N} F_i\), the dynamic stiffness of the pile group for the horizontal mode is simply

\[
K_{X}^{[G]} = K_{X}^{[G]} \sum_{i=1}^{N} \sum_{j=1}^{N} \varepsilon_{X(i,j)}
\] (8)

The vertical dynamic stiffness of the group is also given by an equivalent analysis where index \(X\) is replaced by \(Z\). Accordingly

\[
K_{Z}^{[G]} = K_{Z}^{[G]} \sum_{i=1}^{N} \sum_{j=1}^{N} \varepsilon_{Z(i,j)}
\] (9)

where the \(\varepsilon_{Z(i,j)}\) = inverse of matrix \(\alpha_{Z(i,j)}\) obtained from (2).

The rocking group-dynamic stiffness can be derived by an analysis similar to the one previously presented (Dobry and Gazetas 1988; Makris et al. 1993)

\[
K_{R}^{[G]} = K_{Z}^{[G]} \sum_{i=1}^{N} x_i \sum_{j=1}^{N} x_j \varepsilon_{Z(i,j)}
\] (10)

where \(x_i\) = distance of pile \(i\) from the axis about which the rotation occurs.

For the cross-horizontal-rocking interaction factors it has been found (Gazetas et al. 1991) that the following approximation proposed by Randolph (1977) for static loaded piles, is also valid for dynamic loads:

\[
\alpha_{XR(i,j)} \approx \alpha_{Z(i,j)}^2
\] (11)

Accordingly, the cross-horizontal-rocking group stiffness is

\[
K_{XR}^{[G]} = K_{XR}^{[G]} \sum_{i=1}^{N} \sum_{j=1}^{N} \varepsilon_{XR(i,j)}
\] (12)

where \(\varepsilon_{XR(i,j)}\) = element of the inverse of matrix \(\alpha_{XR(i,j)}\) given by (11).

**Equation of Motion and Solution:** Step 3

The degrees of freedom of the structural model depicted in Fig. 4 are \((u_i; v_i; h\theta_i; u_j; v_j;\) and \(h\theta_j)\). With the linear range and for small rotations,
the equations of motion of the system subjected to a ground acceleration are as follow:

**Horizontal equilibrium of the superstructure**

\[
m_s(\ddot{u}_g + \ddot{u}_f + h\dot{\theta}_f + \ddot{u}_s) + C^s_\chi \dot{u}_s + K^s_\chi u_s + K^s_{\chi R}(\theta_s - \theta_f) = 0
\]  
(13)

**Vertical equilibrium of the superstructure**

\[
m_s(\ddot{v}_g + \ddot{v}_f + \ddot{v}_s) + C^z_\chi \dot{v}_s + K^z_\chi v_s = 0
\]  
(14)

**Rotational equilibrium of the superstructure**

\[
I_s \ddot{\theta}_s = -K^s_\chi (\theta_s - \theta_f) - K^s_{\chi R} u_s
\]  
(15)

**Horizontal equilibrium of the entire system**

\[
m_s(\ddot{u}_g + \ddot{u}_f + h\dot{\theta}_f + \ddot{u}_s) + m_f(\ddot{u}_g + \ddot{u}_f) = -P^F_\chi
\]  
(16)

**Vertical equilibrium of the entire system**

\[
m_s(\ddot{v}_g + \ddot{v}_f + \ddot{v}_s) + m_f(\ddot{v}_g + \ddot{v}_f) = -P^F_z
\]  
(17)

**Rotational equilibrium of the entire system**

\[
I_f \ddot{\theta}_f - K^s_\chi (\theta_s - \theta_f) - K^s_{\chi R} u_s + m_s(\ddot{u}_g + \ddot{u}_f + h\dot{\theta}_f + \ddot{u}_s) = -M^F_R
\]  
(18)

where \(P^F_\chi\), \(P^F_z\), and \(M^F_R\) = horizontal, vertical, and rocking reactions of the pile foundation.

The linearity of the governing differential equations allows for solution in the frequency domain. Applying the Fourier transform to the system of (13)–(18) the equations of motion can be written in a matrix form

\[
[S(\omega)]\{U(\omega)\} = \{F(\omega)\}
\]  
(19)

in which \(\{U(\omega)\} = \text{Fourier transform of the response vector } (u_s, v_s, h\theta_s, u_f, v_f, h\theta_f)\); \(\{F(\omega)\}\) is the Fourier transform of the excitation vector \((-\ddot{u}_g, \ddot{v}_g, 0, -(1 + \mu)\ddot{u}_s, -(1 + \mu)\ddot{v}_s, -\ddot{u}_f)\); with \(\mu = m_f/m_s\); and \([S]\) = frequency dependent dynamic stiffness matrix of the soil-pile-foundation superstructure system. The Fourier transform of the reactions from the pile foundation are given by

\[
F\{P^F_\chi(t)\} = K^F_\chi(\omega) u_f(\omega) + K^F_{\chi R}(\omega) \theta_f(\omega)
\]  
(20)

\[
F\{P^F_z(t)\} = K^F_z(\omega) v_f(\omega)
\]  
(21)

\[
F\{M^F_R(t)\} = K^F_{\chi R}(\omega) u_f(\omega) + K^F_{\chi R}(\omega) \theta_f(\omega)
\]  
(22)

where \(K^F_\chi\), \(K^F_z\), \(K^F_{\chi R}\), and \(K^F_{\chi R} = K^F_{\chi R}\) = horizontal, vertical, rocking, and cross-horizontal-rocking impedances of the pile-foundation, respectively.

The parameters appearing in the dynamic stiffness matrix \([S]\) are

\[
\Omega_{\chi s} = \left(\frac{K^s_\chi}{m_s}\right)^{1/2}; \; \Omega_{\chi z} = \left(\frac{K^s_z}{m_s}\right)^{1/2}; \; \Omega_{\chi R} = \left(\frac{K^s_{\chi R}}{m_s h^2}\right)^{1/2}; \; \Omega_{\chi R s} = \left(\frac{K^s_{\chi R}}{m_s h}\right)^{1/2}
\]  
(23)

\[
\xi_{\chi s} = \frac{C^s_\chi}{2m_s \Omega_{\chi s}}; \; \xi_{\chi z} = \frac{C^s_z}{2m_s \Omega_{\chi z}}
\]  
(24)
\[
\Omega_{xf} = \left( \frac{K_x^{(f)}}{m_x} \right)^{1/2}; \quad \Omega_{zf} = \left( \frac{K_z^{(f)}}{m_z} \right)^{1/2}; \quad \Omega_{rf} = \left( \frac{K_{r}^{(f)}}{m_r h^2} \right)^{1/2}; \quad \Omega_{xrf} = \left( \frac{K_{xrf}}{m_x h} \right)^{1/2}
\]

(25)

\[
\rho_s = \left( \frac{l_s}{m_s} \right)^{1/2}; \quad \rho_f = \left( \frac{l_f}{m_f} \right)^{1/2}
\]

(26)

The system response in the time domain is obtained by the inverse Fourier transform of the response vector \{U(\omega)\} (Clough and Penzien 1993)

\[
\{U(t)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[ H(\omega) \right] \{F(\omega)\} e^{i\omega t} \, d\omega
\]

(27)

where \([H(\omega)] = \text{inverse of matrix} [S(\omega)]\). Relative velocities and acceleration responses are obtained from (27) after having premultiplied the matrix \([H(\omega)]\) by \(i\omega\) and \(-\omega^2\), respectively. Numerical solutions are then derived by the discrete Fourier transform (DFT) method.

**APPLICATION TO PAINTER STREET BRIDGE**

**Superstructure Parameters**

Even for a simplified model like the one presented in the previous section, several uncertainties arise in determining its parameters. Such uncertainties refer to the structural stiffness, structural damping, mass distribution, foundation stiffness, and pier-foundation-soil interaction. The model presented here is a simplification of the stick model discussed by Maroney et al. (1990), where the longitudinal and torsional modes of the stick model are not considered. For the stick model the first transverse period of the superstructure was determined by iteration and reported to be 0.28 s during the earlier (1982–87) seismic event, which had produced less intense shaking of the bridge than the Petrolia earthquake. Accordingly, following our notation \(\Omega_{x0} = 2\pi/0.28 s = 22.4 \text{ rad/s}\). The calibrated (backfigured) horizontal stiffness of the superstructure is reported to be by Maroney et al. (1990) \(K_x = 39,000 \text{ kips/ft} = 5.69 \times 10^5 \text{ kN/m}\). From (25) the mass of the superstructure is \(m_x = 1,130 \text{ Mg}\). The value of \(m_x\) was also estimated by taking the weight of half the deck plus the weight of the half-top part of the piers and the computed value was of the order of 1,000 Mg, which is in agreement with the previously mentioned value. Furthermore, Maroney et al. (1990) reported that, from stress-strain laboratory tests on core samples from existing bridges, the Young's modulus of the concrete was \(E_c = 3,800 \text{ ksi} = 2.6 \times 10^7 \text{ kN/m}^2\). This value is approximately 80% less than the value of \(E_c\) resulting from empirical expressions. Under the stronger shaking of the Petrolia earthquake more cracking is expected to have occurred, and thus the value of \(E_c\) should be further reduced in an equivalent linear dynamic analysis. In this study the value of \(E_c\) reported by Maroney et al. (1990) is reduced by 15%. With this reduction, \(E_c \approx 22 \text{ GPa}\), which is a generally accepted value for moderately cracked concrete. Based on this assumption, the vertical stiffness of the bridge pier, \(K_z\), can be approximated by

\[
K_z = \frac{E_c A_c}{h_c} \approx \frac{(2.2 \times 10^7 \text{ kN/m}^2)(1.19 \text{ m}^2)}{(6 \text{ m})} \approx 4.4 \times 10^6 \text{ kN/m}
\]

(28)

where \(A_c\) and \(h_c\) = bottom cross-section area and net-height of the pier.
The vertical stiffness of the superstructure is approximately
\[ K^v_Z = 2K^c_Z \approx 8.8 \times 10^6 \text{ kN/m} \]  \hspace{1cm} (29)
and from (23) \( \Omega_{Zc} = 96 \text{ rad/s} \). If \( l = \) projection to the N-S axis of the center-to-center distance between the two piers \((l = 9.5 \text{ m})\), the rotational stiffness of the superstructure can be approximated by
\[ K^r_Z = \frac{K^c_Z l^2}{2} \approx (4.4 \times 10^6 \text{ kN/m})(90 \text{ m})^2 \approx 2.0 \times 10^8 \text{ kNm/rad} \]  \hspace{1cm} (30)
and from (23) \( \Omega_{Ks} \approx 60 \text{ rad/s} \). The moment of inertia of the deck is estimated to be \( I_s = 20,000 \text{ Mgm}^2 \) and from (26) the radius of gyration of the superstructure is \( \rho_s = 4.2 \text{ m} \). It should be mentioned that sensitivity studies showed that the deck response was not altered by varying the modulus of concrete \( 20 \text{ GPa} < E_c < 26 \text{ GPa} \). Rather, the deck response is much more sensitive to the values of the foundation stiffness.

**Soil Profile and Foundation Parameters**

Prior to construction a geotechnical exploration at the location of the piers was conducted. With Standard Penetration Test (SPT) measurements from the ground surface down to a depth of about 10 m, moderately stiff/dense soil layers were identified, consisting of clayey sand, silty sand, sandy silt, and gravelly sand. STP blowcounts varied from 8 near the surface to 34 at 10 m depth. The underlying stratum was a very dense gravelly and silty sand, with blowcounts exceeding 100 blows/ft.

Recently, a geophysical exploration was conducted by Heuze & Swift (1991). Six so-called seismic refraction surveys have been reported, along four lines parallel to the highway. Evidently, the differences in the S-wave velocities and shear moduli among the soil profiles below the location of the lines are rather substantial, given that they are located 20–30 m apart from each other. For instance, the resulting low-strain shear modulus from the data along line 2 \((G_s = 100 \text{ MPa})\) has twice the value of that resulting from the data along line 1. It is quite possible that some of these differences reflect merely inadequacies (general and specific) of the seismic refraction technique. Herein, our dynamic analysis of the pile-foundation–bridge system is based on the values extracted from the data along line 2, which is adjacent to the pier \((V_s \approx 250 \text{ m/s, } \rho_s \approx 1.6 \text{ Mg/m}^3, \nu_s = 0.48)\).

Closed-form expressions for the static stiffnesses of single pile have been derived by fitting finite-element results of the static problem (Gazetas 1984). The accuracy of these expressions has been verified by comparing their results with the solution of Blaney et al. (1976) for lateral and vertical pile motion in homogeneous soil, and the solution of Randolph (1981) for lateral pile motion in nonhomogeneous soil with modulus proportional to depth. Using the expressions derived by Gazetas, the soil data along line 2 \((G_s = 100 \text{ MPa, } \nu_s = 0.48)\); pile diameter, \(d = 0.36 \text{ m} \); pile length, \(L = 7.62 \text{ m} \); and Young’s modulus of pile \(E_p = 22,000 \text{ MPa} \), the static stiffnesses of the single fixed-head pile are approximated by

\[ K^{[1]}_X \approx E_p d \left( \frac{E_p}{E_s} \right)^{0.21} \approx 260 \text{ MN/m} \]  \hspace{1cm} (31)

\[ K^{[1]}_Z \approx 1.9G_s d \left( \frac{L}{d} \right)^{2/3} \approx 520 \text{ MN/m} \]  \hspace{1cm} (32)
\[ K_{R}^{[1]} \approx 0.15E_s d^3 \left( \frac{E_p}{E_s} \right)^{0.75} \approx 50 \text{ MNm/rad} \]  
(33)

\[ K_{XR}^{[1]} \approx -0.22E_s d^2 \left( \frac{E_p}{E_s} \right)^{0.5} \approx -75 \text{ MNm/rad} \]  
(34)

The horizontal, \( K_{X}^{[1]} \), and vertical, \( K_{Z}^{[1]} \), static stiffnesses of the single pile are also computed with the procedure developed by Trochanis et al. (1991). The procedure is based on one-dimensional analysis, which utilizes a realistic hysteretic model that has been calibrated using a three-dimensional finite-element analysis of the soil-pile system. This procedure was originally developed to produce load control force-displacement curves. We modified this procedure for displacement control force-displacement curves. Fig. 5 (top plot) plots the horizontal head force versus horizontal head displacement. The resulting horizontal static stiffness of the pile is the slope of the force-displacement curve plotted in Fig. 5, which at small deflections has a value of approximately 200 MN/m. This value is indeed close to the value given by (31). The same computer program developed by Trochanis et al. was used to produce the bottom plot in Fig. 5, which plots the vertical head-force versus the head-settlement of the single pile. The initial stiffness provided by this curve is approximately 450 MN/m, a value that is also very close to the result of (32).

![Graph showing force-displacement curves](image)

**FIG. 5.** Computed Head-Force-Displacement Curves for Single Pile \( (d = 0.36 \text{ m}; L = 7.62 \text{ m}; G_s = 100 \text{ MPa}; s_u = 400 \text{ kPa}; \nu_s = 0.48) \)
FIG. 6. Storage (Real Part) and Loss (Imaginary Part) Stiffness Factors of Fourby-Five Pile Group with Rigid Pile Cap for Horizontal (Top) and Vertical (Bottom) Motions as Function of Dimensionless Frequency with $E_p/E_v = 75$; $\rho_p/\rho_v = 1.5$; $\beta = 0.05$; and $v_s = 0.48$ in a Homogeneous Half Space

All the aforementioned pile-stiffness values have been computed using the small-strain value $G_v (\gamma_s < 10^{-5}) = G_{\text{max}} = 100$ MPa. However, under the Petrolia seismic excitation the level of soil strain is estimated to have been as large as $\gamma_s \approx 10^{-3}$ and occasionally reaching the neighborhood of $\gamma_s = 10^{-2}$. At this strain level the soil-shear modulus is substantially reduced and can be as low as five times less than the small-strain value $G_{\text{max}}$ (Tatsuoka et al. 1979). At this point, we select as average realistic values for the horizontal, vertical, and cross-horizontal-rocking static stiffnesses of the single pile to be $K_x^{\text{||}} = 65$ MN/m; $K_Z^{\perp\perp} = 200$ MN/m; $K_X^{\perp\perp} = 20$ MN/rad; and the rocking stiffness of the individual piles is neglected since it is negligible compared to the large rotational stiffness of the foundation system resulting from the axial vibration of piles. The aforementioned stiffnesses are approximately two to four times smaller than the values obtained with the small-strain value of the soil shear modulus [(31) to (34)]. The reason why the horizontal and cross-horizontal rocking stiffnesses are reduced more than the vertical stiffness is because the soil strains near the soil surface that
primarily influence the horizontal pile motion are larger than the strains at larger depths on which the vertical stiffness depends.

Using the procedure described earlier (step 2), the group stiffnesses are computed with (8)–(10) and (12). For instance, the horizontal and vertical dynamic stiffness coefficients for the 4 x 5 pile group are plotted in Fig. 6 as a function of the dimensionless frequency $a_0 = \omega_0/\nu$. Note that the normalized value of the real part at the zero frequency limit reaches the value of 0.19 for the horizontal and 0.16 for the vertical mode. These values are close to 1/5, which is the value that one obtains using the flexibility ratios provided in classical foundation textbooks (Fleming et al. 1984). The resulting static stiffnesses for the 4 x 5 group are $K_{X}^{4\times5} \approx 250$ MN/m, $K_{Z}^{4\times5} \approx 600$ MN/m; and $K_{XR}^{4\times5} \approx 300$ MN/rad.

To compute the foundation stiffnesses of the Painter Street Bridge, it is
assumed that the motion of a pile belonging in one pile group does not affect the motion of a pile belonging to the other pile group. The minimum distance between two piles belonging to different pile groups is $S = 8 \text{ m} = 26 \text{ ft}$, resulting in an $S/d > 22$. Although for such a high value of $S/d$, no interaction is expected between two piles, the motion of the entire pile group, which has an equivalent diameter $d_e \approx 4.6 \text{ m}$ might influence the motion of the neighbor pile group since $l/d_e \approx 2.5$. Nevertheless, for lack of other evidence, it is assumed that no interaction occurs. To this end, the horizontal, vertical, rocking, and cross-horizontal-rocking stiffnesses of the foundation of the Painter Street Bridge are estimated to be

$$K_x^{[F]} = 2K_x^{[4 \times 5]} \approx 500 \text{ MN/m}$$  \hspace{1cm} (35)$$

$$K_Z^{[F]} \approx 2K_Z^{[4 \times 5]} \approx 1,200 \text{ MN/m}$$  \hspace{1cm} (36)$$
FIG. 9. Comparison of Recorded N-S Deck Acceleration- and Drift-Time History: (A) Prediction of Response when Frequency Dependence of Pile-Foundation Impedances Is Considered; (B) Prediction of Response when Pile-Foundation Impedances Are Computed at Predominant Frequency of Input; (C) Prediction of Response when Pile Foundation Is Assumed Fixed

\[
K_{R}^{(f)} \approx \frac{K_{Z}^{(4 \times 5)}I_{2}}{2} \approx 27,000 \text{ MNm/rad} \tag{37}
\]

\[
K_{XR}^{(f)} \approx 2K_{XR}^{(4 \times 5)} \approx 600 \text{ MNm/rad} \tag{38}
\]

For the preceding values and an approximate value of \( h = 7 \text{ m} \), (25) gives \( \Omega_{Xf} \approx 21 \text{ rad/s} \); \( \Omega_{ZF} \approx 32.5 \text{ rad/s} \); \( \Omega_{RF} \approx 22 \text{ rad/s} \); and \( \Omega_{XR} \approx 9 \text{ rad/s} \). The mass of the foundation was estimated to be \( m_{f} \approx 225 \text{ Mg} \), and the moment of inertia \( I_{r} \approx 8,000 \text{ Mgm}^{2} \). From (26) \( p_{r} = 5.96 \text{ m} \) and \( \mu = m_{f}/m_{s} = 0.2 \).

ANALYTICAL PREDICTION OF RESPONSE

The response of the bridge-foundation system is computed using (27) and compared against the recorded motion. Three different predictions are shown.
Prediction (A) is the result when the entire frequency dependence of the foundation impedance is considered. Prediction (B) is the result when the stiffness and damping of the foundation are computed at the predominant frequency of the input motion \( f_p = 2.32 \text{ Hz}; \omega_p = 14.57 \text{ rad/s} \) obtained from the Fourier spectrum of channel 14. At this frequency the horizontal and vertical dynamic stiffness coefficients are \( k_{x_1} (14.57) + i k_{x_2} (14.57) = 0.970 + i 0.331; k_{z_1} (14.57) + i k_{z_2} (14.57) = 0.903 + i 0.545 \) respectively. Prediction (C) is the result when the foundation is considered as a fixed, monolithic support.

Fig. 7 presents the horizontal (N-S) motion of the pile cap of the bridge foundation. The results of prediction (A) are indeed in very good agreement with the records. Notice that the peak values of both acceleration and displacement are very accurately predicted. The result for the displacement history from prediction (B) are also in very good agreement with the records and the acceleration history is somehow underestimated. Similar prediction are offered from prediction (C) where the acceleration history is now somehow overestimated.

Fig. 8 presents the vertical motion of the pile cap. Again the results of prediction (A) are most favorable for the peak acceleration values; whereas, the total displacement histories of the three predictions are almost identical.

Finally, Fig. 9 presents the horizontal (N-S) deck acceleration and the deck drift relative to the pile-cap level. The results of prediction (A) for the deck drift are good, but the acceleration is underestimated by approximately 30%. Several reasons might be responsible for this discrepancy between recorded and predicted motion, such as neglect of the torsional motion of the bridge about the vertical axis, yielding of bridge pier, yielding of pile heads, and so forth. Such effects are not captured with this simple model. For instance, we noticed that the deck-acceleration response is very sensitive to the value of the cross-horizontal-rocking stiffness of the pile foundation. As the absolute value of the cross-horizontal-rocking stiffness decreases, the predicted deck acceleration becomes more and more pronounced, reaching the recorded values when the cross-horizontal-rocking stiffness is assumed zero. Of course yielding of the pile heads will reduce the value of the cross-horizontal-rocking stiffness. Nevertheless, if one decides to compute the bridge response by neglecting the moment transmitted at the pile heads, the entire analysis should be redone, starting from the interaction factors given by (1), which have to be modified for free-head piles. The results offered by prediction (B) are less favorable, since the response is underestimated by 60%. An interesting result of this study is prediction (C), where the results offered are erroneous. The acceleration of the deck is substantially overestimated (more than 100%), and high frequencies are present. It is a notable example where poor modeling of the foundation affects drastically the response of the superstructure. The origin of this poor prediction is the neglect of the foundation damping. By assuming a fixed, monolithic foundation, no energy dissipation is allowed through the foundation (zero radiation damping), and all the induced seismic energy is trapped into the superstructure and is eventually dissipated only by structural damping.

Another interesting observation is that the ratio of the deck drift to the height of the bridge pier is of the order of \( \delta_r \approx 1\% \). Although there is no precise value of that ratio that indicates level of nonlinearity in the structural response, structures with moderate ductility are expected to experience some nonlinear behavior for \( \delta_r \approx 1\% \).
CONCLUSIONS

A simple integrated procedure has been presented to analyze the problem of soil-foundation-superstructure interaction, using the available theories for the computation of the dynamic impedances and kinematic-seismic-response factors of pile foundations. The procedure utilizes a simple structural model, which can be easily refined and expanded in order to account for more complex structural response, such as torsional motion of the deck, input motion at the end abutments, and so forth. Despite the simplicity of the structural model, the presented procedure gives very encouraging results for the response of the Painter Street Bridge located in Rio Dell, California. The predicted response of the bridge-foundation system is in good agreement with recorded motions. It was found that realistic modeling of the foundation affects drastically the prediction of the superstructural response. Nevertheless, the deck acceleration response was underestimated by 30%. This observation suggests that, for relative strong motions like the Petrolia Earthquake, a nonlinear analysis of the problem could be more realistic.

ACKNOWLEDGMENTS

Partial financial support for this project has been provided by Shimizu Corporation, Japan (Grant No. NCEER/RF150-7014A to NCEER), and by the Federal Highway Administration (Grant DTFH61-92-C-00106 to NCEER).

APPENDIX. REFERENCES


