Kinematic seismic response and bending of free-head piles in layered soil

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The paper studies the kinematic response of freehead piles. Such pile deformation has triggered structural damage in many strong earthquakes. In this Paper dimensionless parametric graphs for pile bending moments are presented which pertain to characteristic two-layer soil profiles. The results are derived by using an existing rigorous dynamic finite-element code, and by implementing a realistic beam-on-dynamic-Winkler-foundation formulation specifically developed for the kinematic response of piles in layered soil. The Winkler model is shown to reproduce quantitatively even detailed trends observed in the finite-element results; a simple analytical expression is thereby developed for estimating the Winkler stiffness in terms of the local soil Young's modulus and key dimensionless pile/ soil parameters. The study concludes that even relatively flexible piles may not exactly experience the wavy and abruptly changing ground deformation of the free field. The critical region of pile distress due to such kinematic loading is shown to be at or near the interface between alternating soft and stiff soil layers. The magnitude of the bending moment at such critical interface locations depends mainly on the stiffness contrast of the two layers through which the pile penetrates, the excitation frequency and the relative rigidity of the pile. A constraining cap may exert an important effect on such kinematic deformations.

KEYWORDS: dynamics; earthquakes; numerical modelling and analysis; piles; soil-structure interaction.

INTRODUCTION

Pile distress and failure during seismic shaking, although difficult to observe in post-earthquake site investigations, have been well documented. For example, Mizuno (1987) has reported on 28 cases involving seismic pile failures in Japan, EEFIT (1986) has described cases of pile extrusion in Mexico City during the 1985 earthquake,

L'article étudie la réponse cinématique de pieux à tête-libre. La déformation de ces pieux a en effet occasionné des endommagements structuraux lors de nombreux tremblements de terre de forte amplitude. L'article propose des graphiques à paramètres adimensionnels permettant de calculer le moment fléchissant des pieux et de caractériser des sols bicouches. Les résultats sont calculés à l'aide d'un code dynamique d'éléments finis, préexistant et rigoureux, et d'une formulation réaliste de "Beam-on-Dynamic-Winkler-Foundation" spécialement mise en oeuvre pour la réponse cinématique des pieux dans des sols stratifiés. Le modèle de Winkler permet de reproduire les tendances, même très détaillées, des résultats obtenus par éléments finis. Une expression analytique simple est alors développée. Elle permet d'estimer la raideur de Winkler en terme de module de Young local du sol et de paramètres clés adimensionnel pieu/sol. L'étude montre que même les pieux relativement flexibles ne supportent pas totalement une déformation rapide et odulante du terrain. La zone critique des pieux sinistrés résultant d'un tel chargement cinématique se trouve au niveau ou à proximité de l'interface sol souple/sol rigide. L'amplitude du moment fléchissant au niveau de ces interfaces critiques dépend principalement du contraste de raideur existant entre les deux couches traversées par le pieu, de la fréquence d'excitation et de la rigidité relative du pieu. Un capuchon de mise sous contrainte pourrait avoir un effet important sur de telles déformations cinématiques.

CNEL-ENEL (1976) has documented pile ruptures under two bridges in the Friuli (Italy) 1976 earthquake and Ross, Seed & Migliaccio (1969) have described numerous failures of piles supporting bridge and harbour facilities in the 1964 Alaska earthquake.

Mizuno (1987), in summarizing the Japanese experience with regard to the likely causes and different types of pile failure, has concluded that many of the failures arose from the transmission onto the foundation of large inertia forces/ moments developing in the superstructure: such failures take the form of either shear/bending cracking and rupturing beneath the head of the

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pile or of the ultimate tension capacity of the soil-pile-cap system being exceeded. Liquefaction-induced failures have also been frequent and spectacular. However, in several cases the location of pile failure was too deep to be caused by loading from the top (due to structural inertia), while liquefaction could not possibly have occurred; damage was in fact associated with the presence of discontinuities in strength and stiffness of the soil profile. The most likely cause is the relatively large curvatures imposed by the surrounding soil as it deforms while excited by up and down (after reflection) propagating seismic waves.

This mode of deformation and potential failure has not received proper attention: in fact, engineers usually ignore the problem altogether and design the piled foundation merely against head loading. However, some theoretical work has been published on the kinematic response of piles (Penzien, 1970; Tajimi, 1969, 1977; Kagawa & Kraft, 1981; Takemiya & Yamada, 1981; Kobori, Minai & Baba, 1981; Wolf & Von Arx, 1982; Flores-Berrones & Whitman, 1982; Kaynia & Kausel, 1982; Gazetas, 1984; Barghouthi, 1984; Dennehy & Gazetas, 1985; Tazoh, Wakahara, Shimizu & Matsuzaki (1988); Nogami, Jones & Mosher, 1991; Ahmad & Mamoon, 1991; Masavuki & Shoichi, 1991). A comprehensive survey of the dynamic and seismic response of piles has been presented by Novak (1991). Moreover, recent seismic codes and seismic guidelines have recognized the importance of this type of loading (AASHTO, 1983; JSCE, 1988; AFPS, 1990; Eurocode EC8, 1990). For example, the first draft of Part 5 of the Eurocode states that: 'Piles shall be designed for the following two loading conditions:

- (a) inertia forces on the superstructure transmitted onto the heads of the piles in the form of an axial force, a horizontal force and a moment... in determining displacements and rotations resulting from these forces, the soil is considered as deforming only due to the transmitted actions....
- (b) soil deformations arising from the passage of seismic waves which impose curvatures and thereby lateral strain on the piles along their whole length.... such kinematic loading may be particularly large at interfaces of soil layers with sharply differing shear moduli. The design must ensure that no 'plastic hinge' develops at such locations....'

While there is ample geotechnical experience of carrying out the equivalent static analysis for the inertial loading (type (a)), no specific method is proposed (let alone required) in EC8 or the other codes referred to in this Paper to predict defor-

mations and bending moments from the kinematic loading (type (b)). Moreover, a search of the literature cited above shows that published information on kinematic bending moments (rather than pile-head deflexions) is so limited, even for the simplest case of a homogeneous soil profile, that the engineer cannot readily assess even their order of magnitude. Recourse to sophisticated methods that are not widely available is a necessary but unattractive alternative. This Paper is intended to bridge the apparent gap in both theory and practice of the seismic analysis/design in two ways.

First, an extensive parametric study is presented on the kinematic response of a single freehead pile to vertically-incident harmonic shear waves (S-waves). The study, conducted using an expanded version of the dynamic finite element formulation developed by Blaney, Kausel & Roesset (1976), concentrates on a two-layer profile which can represent two characteristic cases: a stiff crust underlain by a softer layer, and a soft surficial layer underlain by a stiff soil stratum into which the pile is embedded. The results of the study are presented in dimensionless graphs, which are useful not only for development of an improved understanding of the mechanics of the problem and checking of the accuracy of less rigorous solutions, but also for preliminary design estimates.

Second, a beam-on-dynamic-Winkler-foundation (BDWF) formulation is developed that can be readily used in practical seismic design of piles in layered profiles. Described through rationallyderived 'springs' and frequency-dependent 'dashpots', this BDWF model is shown to accord with the rigorous finite element (FE) predictions of both deflexions and bending moments: maximum deviation of 15% under the most adverse conditions of geometry, soil parameters and excitation frequency. The BDWF spring stiffness is evaluated using a proposed simple analytical expression in terms of key dimensionless problem parameters. The merits of the proposed model include the familiarity of geotechnical engineers with (linear and non-linear) beam-on-Winklerfoundation-type models and its clear computational advantage (a difference of at least a factor of 100) over the rigorous continuum-type solutions.

PROBLEM DEFINITION AND PARAMETRIC RESULTS

The system studied refers to an end-bearing pile embedded in a two-layer soil deposit (Fig. 1), underlain by rigid bedrock and subjected to vertically propagating S-waves. Such waves produce



Fig. 1. Two-layer soil profiles under study

horizontal harmonic motion

 $u_{\mathbf{g}}(t) = U_{\mathbf{g}} \, \mathrm{e}^{i\omega t} \tag{1}$

at the bottom of the lower layer.

The soil is assumed to be a linearly-hysteretic solid with Young's modulus E_a or E_b , damping ratio $\beta_a = \beta_b = 10\%$ (appropriate for moderately strong shaking), mass density $\rho_a = \rho_b$, and Poisson's ratio $v_a = v_b = 0.40$. The pile is a linearly-hysteretic beam with Young's modulus $E_{\rm p}$, bending moment of inertia $I_{\rm p}$, damping ratio $\beta_{\rm p} = 5\%$ and mass density $\rho_{\rm p} = 1.60 \rho_{\rm a}$. The Bernoulli assumption for a beam (plane sections remain plane and perpendicular to its neutral axis) results in the horizontal pile displacement $u_{n}(z, t)$ being the only independent variable of pile deformation. To reduce the required number of analyses (without loss of insight), only the following crucial dimensionless parameters are varied: the pile-to-soil stiffness ratio E_p/E_a , the ratio of the S-wave velocities $V_{\rm b}/V_{\rm a}$ of the two soil layers, the pile slenderness ratio L/d, the ratio of the thicknesses of the soil layers H_a/H_b , and the ratio ω/ω_1 of the excitation frequency to the fundamental natural frequency of the 'free' (i.e. without piles) soil deposit in vertical S-waves.

The sensitivity of bending moments to variation in the values of Poisson's ratios v_a and v_b is also explored. As anticipated in a linear analysis, all computed deformation and stress quantities are proportional to the excitation intensity, expressed by the amplitude of bedrock displacement U_g or the amplitude of bedrock acceleration $\ddot{U}_g = \omega^2 U_g$. Results are presented for normalized bending moments and shear forces

$$\hat{M} = \frac{M}{\rho_{\rm p} d^4 \dot{U}_{\rm g}} \tag{2a}$$

$$\hat{Q} = \frac{Q}{\rho_{\rm p} d^3 \dot{U}_{\rm g}} \tag{2b}$$

while deflexions are normalized by U_g . The previous normalized quantities were obtained by a formal dimensional analysis of the governing differential equations.

The fundamental characteristics of the soil and pile response to harmonic base excitation are investigated by analysis of two series of systems: series 1 (Table 1) is used to study the effect of the soil S-wave velocity ratio V_b/V_a , series 2 (Table 1) is used to study the effect of the pile slenderness ratio L/d. The selected cases, which cover a wide range of possible two-layer profiles, help to investigate the effect of the layer interface on pile bending moments and shear forces for highly contrasting soil properties and various pile slenderness ratios.

Figure 2(a) shows the distribution with depth of the displacement amplitudes in the soil (free field) $U_{\rm ff}$, and the pile, $U_{\rm p}$ (normalized with the common pile and soil displacement at bedrock level) at the natural frequency of the deposit. Only the two extreme S-wave velocity ratio cases are studied: case A (stiff upper crust) and case D (very soft upper layer). As expected, amplification of the motion occurs almost exclusively in the softer layer, while the pile follows the free-field

Case	$V_{\rm b}/V_{\rm a}$	E_p/E_a	H_{a}/H_{b}	L/d	Computed $\omega_1 d/V_a$
Α	0.58	5000	1	20	0.048
В	1	5000	1	20	0.079
С	1.73	5000	1	20	0.116
D	3	5000	1	20	0.141
Е	3	5000	1	10	0.282
G	3	5000	1	40	0.071

Table 1. Series 1 (A-D) and series 2 (E and G) of studied profiles

soil displacement profile only in an average sense. As a result, curvatures sustained by the pile are considerably smaller than those induced on a vertical line in the unperturbed soil. Deviations are evident near the ground surface and at the interface between the upper and lower soil layers. Such deviations merely reflect different pile and soil boundary conditions at these two locations.

Figure 2(b) shows only pile deflexion profiles (normalized with the bedrock displacement amplitude) at the natural frequency of each deposit, for all series 1 profiles. The top-tobottom displacement ratio (amplification) does not change significantly from case to case; differences are limited to the shape of the displacement profile. With highly contrasting stiffness (cases A and D) most of the amplification occurs in the soft layer (i.e. in the lower and upper layers respectively), while the displacement profile achieves a fairly uniform slope in both the homogeneous profile (case B) and the layered profile that exhibits only small differences in relative soil stiffness (case C). The deflexion profiles in Fig. 2(b) should not lead to the conclusion that peak pile displacements near the surface are insensitive to the soil profile characteristics, since the bedrock displacement (used as a normalization factor) corresponds to the natural frequency of the deposit. If, instead, bedrock acceleration is used in the normalization (perhaps a more logical choice for seismic excitation), top deflexions at the natural frequency of the deposit decrease progressively from case A to case D; this is a direct result of the corresponding increase in the fundamental frequency (see Table 1).

The effect of variation of the S-wave velocity ratio and pile slenderness on bending moment and shear force distributions along the piles is shown in Figs 3-5. Fig. 3 plots the normalized amplitudes of bending moment and shear force at the fundamental natural frequency $\omega = \omega_1$ for various S-wave velocity ratios (series 1 cases).



Fig. 2. Comparison of distributions with depth of: (a) free-field (soil) and pile deflexions for profiles A and D; (b) pile deflexions for profiles A, B, C and D



Fig. 3. Distribution with depth of the amplitudes of: (a) bending moment; (b) shear force, at the fundamental natural frequency of the deposit (cases A, B, C and D)

Bending moments are invariably maximal at, or very close to, the interface between the upper and lower soil layers, and zero at the surface and at bedrock level (free-head pile hinged on the bedrock). Moreover, the shape of these moment diagrams reveals that the peak of the M(z) curve near the layer interface becomes sharper with increasing difference in stiffness between the two layers; the peak is flattest with the homogeneous stratum (profile B). The same characteristics are



Fig. 4. Distribution with depth of the amplitudes of: (a) bending moment; (b) shear force, at the fundamental natural frequency of the deposit (cases D, E and G)



Fig. 5. Distribution of bending moment amplitude at the natural frequency of the deposit: solid lines show values computed from the 'exact' pile deflexions; broken lines show values computed from the free-field soil displacements, i.e. on the assumption that piles follow the soil motion exactly

shown in Fig. 4(a), which contrasts bending moment distribution for various pile slenderness ratios (series 2 cases). The very short and hence rigid pile (case E) does not follow the soil displacement curvatures closely; normalized bending moments are smaller than in more flexible piles (cases D and G) while in absolute terms Mincreases approximately in proportion to pile length and to d^3 .

The importance of the soil-pile kinematic interaction is better elucidated with reference to Fig. 5. The 'exact' bending moment distributions at $\omega =$ ω_1 are plotted in solid lines for the two extreme cases studied (A and D, corresponding to a stiff crust and a soft upper layer respectively). The dashed lines show the corresponding bending moment distributions if interaction is neglected, i.e. if pile displacements are assumed to be equal to free-field soil displacements-an assumption often invoked in seismic design practice (see for example Margason & Holloway (1977)). Significant errors evidently could occur in the estimation of bending moments under this assumption. Consequently, soil-to-pile kinematic interplay should not be neglected.

Figure 6 plots the spectrum of maximumalong-the-pile bending moment amplitude as a function of the frequency ratio ω/ω_1 for the series 1 profiles. In most cases studied, the largest maximum value max. $M_{\rm m}(\omega)$ has indeed been found to occur at the fundamental frequency of the deposit. In case A in particular, max. $M_{\rm m}(\omega)$ is associated with a sharp amplification of motion despite the hysteretic damping of the soil (10%). The maximum bending moment decreases rapidly for frequencies higher than ω_1 due to the rapidly increasing radiation damping and the increasing waviness of soil which cannot be followed by the pile. At excitation frequencies lower than the fundamental frequency of the deposit there is little radiation damping, since laterally-spreading waves (which are carriers of radiated energy) are not generated, e.g. Kausel, Roesset & Waas (1975), Dobry, Vicente, O'Rourke & Rocsset (1982), Gazetas (1983), Krishnan, Gazetas & Velez (1983).

However, max. $M_{\rm m}(\omega)$ does not always occur at the fundamental frequency of the deposit: for certain combinations of pile/soil parameters the largest peak can be shown to occur at the second natural frequency. This could have been anticipated, since bending moments in the pile are controlled by two counteracting factors

- (a) the value of the normalized curvature of the pile displacement shape—larger values occur in the higher modes as they are more 'wavy'
- (b) the overall amplitude of the pile displacement profile—larger values occur in the lower modes. The peaks of soil displacement amplification (ratio of top to bottom free-field soil displacements) are in first approximation inversely proportional to 2n - 1, where n is the mode number (e.g. Roesset (1977), Gazetas



Fig. 6. Maximum bending moment amplitude (at the most adverse location along the pile) as a function of the frequency

(1987)); hence the amplification in the second mode would be only one-third of that in the first (fundamental) mode.

The second mechanism usually prevails, and thus peak response occurs at the fundamental mode, but in some cases the first mechanism is dominant and the response is largest at the second natural frequency. Figure 7 shows the depth at which pile bending moment becomes maximum with varying frequency (series 1 profiles). At low frequencies the maximum occurs at, or very close to, the layer interface. At higher frequencies, which can excite effectively higher mode shapes, the location of



Fig. 7. Depth at which maximum bending moment amplitude occurs along the pile as a function of frequency



Fig. 8. (a) Maximum bending moment amplitude (at the most adverse location along the pile); (b) location of the maximum bending moment amplitude along the pile, as a function of frequency



Fig. 9. Effect of soil Poisson's ratio on the maximum bending moment amplitude (at the most adverse location along the pile) as a function of frequency: case D

maximum moment is shifted away from the interface (above or below). However, with an actual earthquake excitation, containing many frequencies, the maximum moment should be expected to be within a two-diameter distance of the interface, in accordance with the design rules of the first draft of Eurocode EC8 (1990). With more flexible piles $(E_p/E_a < 5000)$, this distance from the interface is reduced to one diameter.

Figure 8 plots the spectrum of maximum bending moment amplitude and its location along the pile (series 2 profiles). The largest normalized maximum bending moment $M/(\rho_p d^4 U_g)$ increases with pile slenderness and occurs near the layer interface. Fig. 9 illustrates the effect of soil Poisson's ratios v_a and v_b on the spectrum of maximum-along-the-pile bending moment amplitude as a function of the frequency ratio for case D (soft upper layer). Moments increase slightly with increasing Poisson's ratio. They appear to be more sensitive to the Poisson's ratio of the softer layer, but the overall variation is less than 10% despite the wide range of Poisson's ratios used.

A SIMPLIFIED MODEL FOR THE KINEMATIC RESPONSE

The pile displacement and bending moment profiles presented above elucidate the dynamic interplay of soil and pile response. This interplay is especially noticeable when piles penetrate soil layers with strongly contrasting stiffnesses. However, rigorous analytical tools (such as the FE model with wave-transmitting boundaries employed in the previous analyses), even if available, have well-known limitations when used in seismic design. This is particularly true if seismic analysis using actual or simulated ground motions is to be performed in the frequency domain, since pile response must be computed at a large number of frequencies (of the order of thousands) covering the frequency content of the seismic signal. Therefore, a simplified analytical

model would be quite useful provided that it had been shown to match the rigorous results adequately for a wide range of pile types, soil profiles and excitation frequencies.

The simplified model proposed in the present study satisfies the above requirements. It is based on the BDWF approach, in which the soil is represented by springs and dashpots continuously distributed along the pile length (Fig. 10). This approach has been used extensively to estimate the dynamic impedances of piles in relation to inertial interaction studies, i.e. for dynamic excitation applied to the top of the pile (e.g. Novak (1974), Berger & Pyke (1977), Novak & Aboul-Ella (1978), Bea (1980), Sanchez-Salinero (1982), Dobry et al. (1982)). A few studies have also used Winkler-type models to determine the kinematic deflexion of piles. Penzien (1970) developed a lumped-parameter model of the pile and introduced non-linear springs and dashpots with intuitively-evaluated parameters to represent pile-soil interaction. Flores-Berrones & Whitman (1982) used linear Winkler springs of arbitrarilyassigned stiffness $k = 72s_u$ (where s_u is the undrained shear strength of the soil), ignoring radiation and hysteretic damping, to obtain qualitative estimates of the seismic deflexion of a pile in a homogeneous stratum. Barghouthi (1984) went a step further by utilizing Novak's plane-strain thin-layer solution (Novak, Nogami & Aboul-Ella, 1978) to assign theoretically-sound frequency-dependent spring and dashpot values and to study the response of piles embedded in a homogeneous stratum, under several types of seismic excitation.

The Winkler model developed here differs from those of the studies referred to above in that it is applied to two-layered (rather than homogeneous) deposits, it proposes rational closedform expressions for springs and dashpots based on three-dimensional finite-element results (as opposed to Novak's two-dimensional solution), and it is calibrated for maximum kinematic bending moments (rather than pile-head deflexions). The kinematic response in a homogeneous stratum using closed-form expressions for the springs and dashpots based also on FE results has been studied by Kaynia & Kausel (1980) for sleeved piles and by Dennehy & Gazetas (1985) for sheet piles. The specific details of the proposed BDWF model are as follows.

The soil surrounding the piles is assumed to consist of the free field, where the seismic S-waves propagate vertically, unaffected by the presence of the pile, and an interaction zone where soil motions affect and are affected by the pile. The analysis is performed in two stages, as shown in Fig. 10. In the first stage, the free-field soil motions are computed using a suitable one-



Fig. 10. Proposed BDWF model for a multi-layered soil profile and a free-head pile: the system is excited by vertically-propagating S-waves

dimensional S-wave propagation method. In this study, free-field displacements

$$u_{\rm ff}(z, t) = U_{\rm ff}(z) \exp[\iota(\omega t + \alpha_{\rm ff})]$$
(3)

produced by vertically-incident harmonic S-waves are computed analytically, assuming linear hysteretic soil behaviour. Each layer is characterized by a complex shear wave velocity

$$V_{\rm s}^* = V_{\rm s} \sqrt{(1+2\imath\beta)} \tag{4}$$

where $V_s = \sqrt{(G/\rho)}$ is the actual shear wave velocity and β is the hysteretic damping ratio. (However, within the framework of the developed procedure, equivalent linear and non-linear soil models could also possibly be used to this end.) The details of this stage of the analysis are not given here as they can be found in Schnabel, Lysmer & Seed (1972), Roesset (1977) and Kausel & Roesset (1984).

The second stage of the analysis computes the response of the pile and its adjacent interaction zone, modelled by continuously distributed horizontal springs (of stiffness k_x) and dashpots (of viscosity c_x) excited at their support by the free-field soil displacements $u_{\rm ff}(z, t)$ computed in the first stage of the analysis. At the other end, the springs and dashpots are connected to the pile,

on which they transmit horizontal displacements

$$u_{\rm p}(z, t) = U_{\rm p}(z) \exp[i(\omega t + \alpha_{\rm p})]$$
(5)

and produce bending moments and shear forces. The force (per unit length of pile)-to-displacement ratio of the Winkler medium defines the complexvalued frequency-dependent impedance

$$S_x = k_x + \iota \omega c_x \tag{6}$$

As a first approximation, based on comparative finite-element studies (Gazetas & Dobry, 1984a), the spring stiffness k_x could be considered to be approximately frequency-independent and expressed as a multiple of the local soil Young's modulus E_s

$$k_x \approx \delta E_s$$
 (7)

where δ is a frequency-independent coefficient assumed to be constant (i.e. the same for all layers and independent of depth). The evaluation of δ (in terms of key pile and soil properties), by use of the FE results on bending moments, is one of the main contributions of the present study.

The stiffness parameter c_x in equation (6) represents both radiation and material damping; the former arises from waves originating at the pile perimeter and spreading laterally outward and the latter from hysteretically-dissipated energy in

the soil. The following algebraic expression, based on the work of Roesset & Angelides (1980), Krishnan *et al.* (1983) and Gazetas & Dobry (1984a, 1984b) is used here

$$c_x \approx (c_x)_{\text{radiation}} + (c_x)_{\text{hysteresis}}$$
 (8)

or

$$c_x \approx 2d\rho_s V_s \left[1 + \left(\frac{V_c}{V_s}\right)^{5/4}\right] a_0^{-1/4} + 2k_x \frac{\beta}{\omega} \qquad (9)$$

where $a_0 \equiv \omega d/V_s$ is the dimensionless frequency and V_c is the apparent velocity of the extensioncompression waves, taken as the Lysmer's analogue velocity (introduced by Gazetas & Dobry (1984a, 1984b))

$$V_{\rm c} \approx V_{\rm La} = \frac{3 \cdot 4V_{\rm s}}{\pi (1 - \nu)} \tag{10}$$

at all depths except near the ground surface $(z \le 2.5d)$, where three-dimensional effects arising from the stress-free boundary are better reproduced by use of

$$V_{\rm c} \approx V_{\rm s}$$
 (11)

Furthermore, radiation damping must essentially vanish for excitation frequencies lower than the fundamental frequency in shear of the soil profile, as explained above.

Based on the above, the governing differential equation of the pile response is

$$E_{\rm p}I_{\rm p}\frac{\partial^4 u_{\rm p}}{\partial z^4} + m_{\rm p}\frac{\partial^2 u_{\rm p}}{\partial t^2} = S_{\rm x}(u_{\rm ff} - u_{\rm p})$$
(12)

The solution to equation (12) for a two-layered profile and harmonic wave excitation is outlined in Appendix 1. Evidently, the computed pile response will depend on the chosen value of the Winkler spring parameter δ . The sensitivity of pile deflexions and bending moments to variations in δ is explored in Figs 11–13. The range of values studied ($1 \le \delta \le 4$) is wider than that reported in the literature for pile-head loading (e.g. Vesic (1961), Gazetas & Dobry (1984a)).

Figure 11 demonstrates that seismic pile-head deflexion is rather insensitive to changes in δ , except perhaps at a few frequencies. This is probably because pile deflexions are largely governed by the free-field soil displacements (which of course are independent of δ). In contrast, the maximum bending moments $M_{\rm m}(\omega)$ (Fig. 12) as well as the bending moment and shear force distributions at resonance $M(z, \omega_1)$ and $Q(z, \omega_1)$ (Fig. 13) show some sensitivity to δ .

In earlier studies of dynamic pile response, an optimum value of δ was obtained by matching



Fig. 11. BDWF model predictions of: (a) interaction transfer function (ratio of pile-head to ground-surface displacements); (b) amplitude of pile-top deflexion, as functions of frequency for various values of the coefficient δ : the FE prediction is almost identical to the curve for $\delta = 2.5$; only a few FE points are shown



Fig. 12. BDWF model predictions of the maximum bending moment amplitude (at the most adverse location along the pile) as a function of frequency for various values of the coefficient δ : the FE prediction is almost identical to the curve for $\delta = 2.5$; only a few FE points are shown



Fig. 13. BDWF model predictions of the amplitudes of: (a) bending moment profiles; (b) shear force profiles, at the natural frequency of the deposit for various values of the coefficient δ : the FE prediction is almost identical to the curve for $\delta = 2.5$; only a few FE points are shown

Case	$V_{\rm b}/V_{\rm a}$	$E_{\rm p}/E_{\rm a}$	$H_{\rm a}/H_{\rm b}$	L/d	Computed $\omega_1 d/V_a$
1 2 3 4	0.58	500 5000	1	10 20 40 10	0·0964 0·0482 0·0241 0·0964
5 6				20 40	0·0482 0·0241
7 8 9 10	1.20	1000 10000	3 0·33 1 3	20	0·0916 0·1164 0·1058 0·0913
11 12 13 14 15 16	1.73	500 5000	1	10 20 40 10 20 40	0·2310 0·1155 0·0578 0·2310 0·1155 0·0578
17 18 19 20	3	1000 10000	3 0·33 1 3	20	0·1017 0·2117 0·1415 0·1019
21 22 23 24	6	1000 10000	3 0·33 1 3	20	0·1042 0·2897 0·1535 0·1045

 Table 2.
 Series 3 of studied profiles



Fig. 14. BDWF model coefficient δ values computed by use of equation (13) for typical soil and pile parameters

the values of the static pile-head stiffness computed with the Winkler model and with a rigorous (e.g. dynamic FE) formulation (see for example Roesset & Angelides (1980), Dobry *et al.* (1982), Gazetas & Dobry (1984a)). In view of the insensitivity of pile deflexions to variations in δ and an interest in the assessment of pile distress due to seismic wave propagation, δ in this study was back-figured by matching BDWF and FE values for the maximum-over-depth bending moment at the natural frequency of the deposit $M_{\rm m}(\omega_1)$: this value of δ is referred to below as the optimum value $\delta_{\rm opt}$. For case 12 (Figs 11-13), $\delta_{\rm opt} \approx 2.5$.

The success of the developed BDWF model can be judged from the fact that it also matches the following FE results

- (a) the whole range of maximum-over-depth bending moments $M_m(\omega)$ —as shown in Fig. 12, the achieved agreement is excellent over the whole frequency range of practical interest, and for all parametric cases studied
- (b) the distribution of bending moment and shear force with depth at a particular frequency M(z, ω)—as shown in Fig. 13 for case 12 at ω = ω₁, the agreement is satisfactory

Case	$\underline{M_{\mathrm{m}}(\omega_{1})^{*}}$	δ_{opi}^{\dagger} †	δ_{comp} ‡	$M_{\rm m}(\omega_1)$ §	Error: %
	$ ho_{p}d^{4}\ddot{U}_{g}$			$ ho_{ m p} d^4 \ddot{U}_{ m g}$	
1	709	1.69	2.21	760	7.2
2	1253	2.08	2.41	1282	2.3
3	2088	2.07	2.63	2196	5.1
4	1854	1.89	1.66	1623	-12.4
5	7718	1.50	1.81	8019	3.9
6	13120	1.71	1.97	13320	1.5
7	508	2.14	2.27	515	1.4
8	1113	1.59	1.42	1097	-1.5
9	931	1.00	1.55	1070	15.0
10	1482	1.79	1.70	1433	-3.3
11	86	2.18	2.05	79	-7.7
12	269	2.50	2.24	253	- 5.6
13	656	3.00	2.44	577	-12.0
14	273	1.67	1.54	266	-2.6
15	998	2.35	1.68	860	-13.9
16	3054	2.16	1.83	2825	-7.5
17	1046	1.92	2.17	1075	2.8
18	875	1.28	1.35	896	2.3
19	2949	1.30	1.48	3042	3.2
20	4867	1.56	1.63	4911	-0.9
21	1486	2.06	2.07	1480	-0.4
22	1128	1.00	1.29	1274	13.0
23	3809	0.95	1.42	4193	10.1
24	6730	1.36	1.55	6831	1.5

Table 3. Series 3 profiles: comparison of FE and BDWF results

* Computed maximum moment at resonance (FE method).

† Value of δ required for the BDWF model to match the FE results at resonance.

 \ddagger Value of δ computed from the proposed formula equation (13).

§ Maximum moment prediction at resonance. BDWF model prediction using δ_{comp} values.

|| Percentage error in the BDWF moment as compared with the FE moment.

(c) the pile-head displacement spectrum $U_{n}(0, \omega)$ as shown in Fig. 11, and the distribution of displacements with depth $U_n(z, \omega)$ (not shown here).

The value of δ_{opt} depends on the dimensionless geometric and material parameters of the problem. With the results for the series 3 profiles (Table 2), a multiple regression analysis was performed to derive a closed-form expression for δ_{out} . For simplicity, and in view of the demonstrated low sensitivity of the results to the exact value of δ , the regression coefficients were rounded off to produce the approximate expression

$$\delta_{\rm comp} = \frac{2}{1 - v_{\rm s}^2} \left(\frac{E_{\rm a} d^4}{E_{\rm p} I_{\rm p}} \right)^{1/8} \left(\frac{L}{d} \right)^{1/8} \\ \times \left(\frac{H_{\rm a}}{H_{\rm b}} \right)^{1/12} \left(\frac{V_{\rm a}}{V_{\rm b}} \right)^{1/15}$$
(13)

which for the case of a homogeneous soil and a pile with circular cross-section simplifies to

$$\delta_{\rm comp} \approx \frac{3}{1 - v_{\rm s}^2} \left(\frac{E_{\rm s}}{E_{\rm p}}\right)^{1/8} \left(\frac{L}{d}\right)^{1/8}$$
 (14)

Equation (13) is plotted in Fig. 14 for the case of soil layers of equal thickness. The values of the spring coefficient δ are not very sensitive to variations in soil and pile properties, at least for realistic values of these properties.

Table 3 summarizes the results of assessment of the performance of the BDWF model by use of the procedure suggested above. Despite some differences between δ_{opt} and δ_{comp} , the computed $M_m(\omega_1)$ is within 15% of the optimum value. All other quantities of interest are equally well predicted using δ_{comp} . Note that the percentage errors are the maximum errors over all frequencies and all locations along the pile. The comparison of the peak moment (at resonance) computed by the FE formulation and the BDWF model (using δ_{comp}) is satisfactory, despite the sharp response at this frequency, with deviations $\leq 15\%$ (<7% in most cases).

The proposed relationship for the Winkler spring coefficient (equations (13) and (14)) is reminiscent of the relationship derived by Vesic (1961) for the analogous static problem of an infinitelylong beam (modulus E_p) on the surface of a homogeneous elastic half-space (modulus $E_{\rm s}$). By comparing the 'exact' bending moment distribution with that obtained by the 'subgrade-reaction' (Winkler) model, Vesic proposed the following relationship for the static spring coefficient

$$\delta = \frac{0.65}{1 - v_s^2} \left(\frac{E_s d^4}{E_p I_p} \right)^{1/12}$$
(15)

Equation (15) has the same form as equation (14) and shows, understandably, a weaker dependence of δ on the pile-to-soil stiffness ratio than do equations (13) and (14).

CONCLUSIONS

The kinematic interaction between soil and a free-head pile during seismic excitation consisting of vertically-propagating harmonic S-waves has been shown to be important. The magnitude of the bending moments developed in the pile may be appreciable, especially near interfaces of soil layers with highly contrasting S-wave velocities. Such profiles are quite common: examples include the cases of a stiff overconsolidated clay crust underlain by a softer soil, and a soft surficial layer underlain by a stiff soil stratum. If strong seismic excitation is anticipated, the pile sections near layer interfaces should be designed with the necessary strength and ductility so that their vertical load-carrying capacity is maintained, just as required by Eurocode EC8 (1990).

The parametric graphs shown for the kinematically-induced bending moments fill a gap in the geotechnical-earthquake literature. Such graphs could be readily used in preliminary design calculations, but also help to develop insight into the mechanics of pile-soil kinematic interplay. For more detailed design calculations, a versatile BDWF model has been developed and calibrated. Simple analytical expressions are proposed for estimation of the stiffness of the continuously-distributed Winkler springs, as well as the viscosity of the associated Winkler dashpots, that can reproduce the radiation and hysteretic damping of the system. While the calibration emphasizes bending moments, the BDWF model is shown to predict pile deflexion in accordance with more rigorous FE solutions.

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APPENDIX 1. DYNAMIC WINKLER MODEL FOR PILE RESPONSE TO HARMONIC S-WAVES The harmonic free-field response

$$u_{\rm ff}(z,t) = U_{\rm ff} \exp\left[\iota(\omega t + \alpha_{\rm ff})\right] = U_{\rm ff} \exp\left(\iota\omega t\right) \qquad (16)$$

is determined using well-established wave propagation (soil amplification) methods (Schnabel et al., 1972; Roesset, 1977, etc.). The deflexion of the pile (see Fig. 11)

$$u_{p}(z,t) = U_{p} \exp\left[\iota(\omega t + \alpha_{p})\right] = \hat{U}_{p}(z) \exp\left(\iota\omega t\right)$$
(17)

is then derived from the steady-state solution of equation (12) for each soil (and pile) layer, or from the ordinary differential equation

$$\hat{U}_{pp}^{\prime\prime\prime\prime} - \lambda^4 \hat{U}_{pp} = \alpha \hat{U}_{ff} \tag{18}$$

where

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$$\lambda^{4} = \frac{m_{p}\omega^{2} - S_{x}}{E_{p}I_{p}} \quad \text{and} \quad \alpha = \frac{S_{x}}{E_{p}I_{p}}$$
(19)

Equation (16) has the general solution

$$\hat{U}_{pp}(z) = \begin{bmatrix} e^{-\lambda z} & e^{\lambda z} & e^{-i\lambda z} & e^{i\lambda z} \end{bmatrix} \begin{cases} D_1 \\ D_2 \\ D_3 \\ D_4 \end{cases} + s \hat{U}_{ff}(z) \qquad (20)$$

where $s = \alpha/(q^{*4} - \lambda^4)$ and D_1 , D_2 , D_3 , D_4 are arbitrary constants to be evaluated from the compatibility equations and the boundary conditions. By use of equation (20)

$$\begin{cases} \hat{U}_{pp}(z) \\ \hat{U}'_{pp}(z) \\ \hat{U}''_{pp}(z) \\ \hat{U}''_{pp}(z) \end{cases} = \begin{bmatrix} e^{-\lambda z} & e^{\lambda z} & e^{-i\lambda z} & e^{i\lambda z} \\ -\lambda e^{-\lambda z} & \lambda e^{\lambda z} & -i\lambda e^{-i\lambda z} & i\lambda e^{i\lambda z} \\ \lambda^2 e^{-\lambda z} & \lambda^2 e^{\lambda z} & -\lambda^2 e^{-i\lambda z} & -\lambda^2 e^{i\lambda z} \\ -\lambda^3 e^{-\lambda z} & \lambda^3 e^{\lambda z} & i\lambda^3 e^{-i\lambda z} & -i\lambda^3 e^{i\lambda z} \end{bmatrix} \\ \times \begin{cases} D_1 \\ D_2 \\ D_3 \\ D_4 \end{cases} + s \begin{cases} \hat{U}_{ff}(z) \\ \hat{U}''_{ff}(z) \\ \hat{U}''_{ff}(z) \end{cases}$$
(21)

or concisely, for a pile element in the domain of soil layer j

$$\tilde{U}_{pj}(z) = \tilde{F}_{j}(z) \cdot \tilde{D}_{j} + s_{j} \tilde{U}_{j}(z)$$
(22)

The vector $\tilde{U}_{j}(z)$ is available from the soil amplification solution.

In the case of a multi-layer soil profile with N layers (j = 1, 2, ..., N), equation (21) consists of a set of 4Nequations with 4N arbitrary constants $\tilde{D}_1, \tilde{D}_2, \dots, \tilde{D}_N$. These constants can be evaluated from the compatibility equations and the boundary conditions.

Compatibility equations

At the (N-1) soil layer and pile interfaces, the pile deflexion u_p , rotation θ , moment, M and shear force Q must be continuous. These compatibility requirements can be expressed by the following 4(N-4) equations (for an arbitrary interface j)

$$\tilde{U}_{\mathbf{p}j}(z_j) = \tilde{U}_{\mathbf{p}(j+1)}(z_j) \tag{23}$$

Boundary conditions

At the pile top, in the case of a free-head pile

$$Q(0, t) = M(0, t) = 0$$
(24)

At the pile base, in the case of a pile hinged at the bedrock

$$M(z_N, t) = 0$$
 and $u_p(z_N, t) = u_g(t)$ (25)

A set of 4N equations is thus obtained which can be solved for the constants $\tilde{D}_1, \tilde{D}_2, \dots, \tilde{D}_N$. Once these constants are evaluated, pile displacements, moments, shear forces etc. can be obtained directly from equation (21) since

ent: $U_{pp}(z)$ $\Theta(z) = U'_{pp}(z)$ $M(z) = -E_p I_p U''_{pp}(z)$ $Q(z) = -E_p I_p U''_{pp}(z)$ pile displacement: pile rotation: pile moment: pile shear:

Extension of the above analysis to floating piles (i.e. piles not reaching the bedrock) as well as to other boundary conditions (e.g. piles restrained from rotation at the top or piles fixed at bedrock) is straightforward.

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NOTATION

c_x	coefficient of the distributed winkler
-	dashpots
d	pile diameter
E _s	soil Young's modulus in general
E_{a}, E_{b}, E_{i}	soil layer Young's moduli
E_{p}	pile Young's modulus
$G_{a}, G_{b}, \dot{G}_{i}$	soil layer shear moduli
$H_{\rm a}, H_{\rm b}, H_{\rm i}$	soil layer thicknesses
	pile cross-section area moment of
P	inertia
k.	stiffness of the distributed Winkler
*	springs
L	pile length
$M = M(z, \omega)$	amplitude of bending moment along
())	the pile
$M_{-} = M_{-}(\omega)$	amplitude of maximum bending
	moment along the pile
m	pile mass per unit length
ő	amplitude of shear force along pile
Š	complex stiffness of Winkler-type soil
^o x	resistance
II	amplitude of free-field soil displacement
	amplitude of hedrock displacement
ť ľ	amplitude of bedrock displacement
	amplitude of pile displacement
0 p	free field soil displacement
"ff	hedrock displacement (used as
^u g	excitation)
	nile displacement
	pile displacement
$v_{\rm s}, v_{\rm a}, v_{\rm b}, v_{\rm si}$	of layers a b and i)
_	vartical as ordinate (donth)
Z	denth at which the maximum hending
² m	moment occurs along the pile
	moment occurs along the pile
α_{p}, α_{ff}	phase angles of phe and free-field soll
0000	displacements
<u> к к к. к.</u>	nysteretic damping ratios of soil

- $\beta, \beta_{a}, \beta_{b}, \beta_{i}$ (general, of layers a, b and i)
 - pile damping ratio β_{b}
 - frequency-independent stiffness coefficient of the Winkler springs $\sqrt{(-1)}$ ı

 $\rho_{\rm p}$ pile mass density

 $\rho_{s}, \rho_{a}, \rho_{b}, \rho_{i}$ soil layer mass densities (general, of layers a, b and i)

- ω excitation circular frequency
- ω_1 natural circular frequency of the soil deposit

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