HORIZONTAL DAMPING OF ARBITRARILY SHAPED EMBEDDED FOUNDATIONS

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ABSTRACT: Using numerical results obtained by the boundary element method, the paper develops a simple method involving algebraic formulas and dimensionless parametric charts for estimating the lateral radiation damping coefficients of arbitrarily shaped rigid foundations embedded in an elastic halfspace. Complete as well as partial, symmetric and nonsymmetric types of contact are considered between the vertical sidewalls and the surrounding soil. Valuable insight is gained into the mechanics of radiation damping and conclusions of practical significance are drawn regarding the effects of potential separation/slippage between the walls and the soil near the ground surface. Two examples illustrate the direct applicability of the method while its versatility is shown through an extension to treat a more realistic inhomogeneous soil profile.

INTRODUCTION

This is the second of a two-paper sequence studying the horizontal dynamic response (swaying) of rectangular and arbitrarily-shaped foundations embedded in an elastic halfspace and subjected to harmonic excitation of frequency $\omega$. The first paper (10) presents results for estimating the static stiffnesses, $K_x$ and $K_y$, and the dynamic stiffness coefficients, $k_x = k_x(\omega)$ and $k_y = k_y(\omega)$, of such foundations. The objective of this paper is to provide information for assessing the magnitude of the two radiation damping coefficients, $C_x = C_x(\omega)$ and $C_y = C_y(\omega)$, which represent the spreading of elastic energy carried by waves propagating away from the foundation. Recall that stiffness and damping terms combine as follows to give the dynamic force-displacement ratio (impedance), $P_x/u_x$:

$$\frac{P_x}{u_x} = \frac{P_{x0} \exp(i\omega t)}{u_{x0} \exp(i\omega t)} = K_x k_x(\omega) + i\omega C_x(\omega) \tag{1}$$

$P_y/u_y$ is similarly related to $K_y$, $k_y$ and $C_y$. From Eq. 1, $C_x$ and $C_y$ can be interpreted as the equivalent dashpot coefficients (5,18).

The numerical data presented in this paper has been derived with an efficient and rigorous boundary-element formulation outlined in the first paper (10). Results are again presented in the form of dimensionless parametric graphs, for a wide range of embedment depths, basemat geometries, frequencies, and values of Poisson’s ratio. The height of the foundation sidewall is varied parametrically between the two extremes of no-sidewall and complete-sidewall with height equal to the depth of embedment. Simple algebraic formulas are also developed for predicting

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the radiation damping of foundations having complicated solid basement shapes and nonsymmetric sidewall contact along their perimeter. Such formulas, although calibrated with numerical data available only for a homogeneous halfspace, may be readily extended to encompass some more realistic soil profiles.

In reality, in addition to loss due to radiation, energy is dissipated by hysteretic action in the soil, conveniently expressed through a frequency-independent material damping ratio, \( \beta \). Having estimated the radiation damping coefficients \( C_x \) and \( C_y \) with the help of the diagrams and formulas of this paper, the effect of material damping is incorporated using the correspondence principle of viscoelasticity. The combined radiation-hysteretic dashpot coefficients \( C_x(\beta) \) and \( C_y(\beta) \) can be approximated as:

\[
C_x(\beta) = C_x + \frac{2K_x K_x}{\omega} \beta
\]

(and similarly for \( C_y(\beta) \)), following Lysmer’s interpretation of this principle (14). The stiffness terms are estimated from the results of the companion paper (10). In the sequel, only radiation damping is studied.

**Damping of a Surface Foundation**

A horizontally oscillating surface foundation transmits primarily shear waves into the supporting ground. These waves originate at every point of the soil-basement interface and propagate with velocity \( V_s = \sqrt{G/\rho} \). At very high frequencies (very small wavelengths), all these points on the interface radiate as independent sources and the outward spreading waves tend to cancel each other (destructive interference) in directions other than perpendicular to the piston (2,4). Hence, the radiation phenomenon tends to become one-dimensional, with the waves moving vertically downward. Then, regardless of basement shape, both \( C_{bx} \) and \( C_{by} \) tend to become equal to \( \rho V_s A_b \), where \( A_b \) = the basement area.

At very high frequencies, the two dashpot coefficients can be expressed in the form:

\[
C_{bx} = \tilde{c}_x \rho V_s A_b; \quad C_{by} = \tilde{c}_y \rho V_s A_b
\]

where the dimensionless frequency and shape-dependent parameters

\[
\tilde{c}_x = \tilde{c}_x \left( a_0; \frac{L}{B}, \nu \right); \quad \tilde{c}_y = \tilde{c}_y \left( a_0; \frac{L}{B}, \nu \right)
\]

are shown in Fig. 1. Foundation shapes included in this figure are the circle and the square, rectangles of aspect ratios \( L/B = 2, 4, 6, \) and 10, and the strip. The effect of Poisson’s ratio is shown only for the values of \( \tilde{c}_x \) corresponding to circular and strip basements. The numerical data on which Fig. 1 is based have been obtained by the authors with the aforementioned boundary-element formulation for \( L/B = 1 – 10 \), from Refs. 12 and 19 for the circular footing, and from Refs. 13 and 9 for the strip. These data are consistent (differences within 5%) with the results in Refs. 5 and 21. The following trends are worthy of note in Fig. 1:
FIG. 1.—Charts for the Lateral and Longitudinal Radiation Dashpot Coefficients, \( C_y \) and \( C_x \), of Surface Foundations (Effect of Poisson’s Ratio Shown only on \( C_y \) for \( L/B = 1 \) and \( \infty \)).

1. The dimensionless damping parameter for oscillation in the long direction, \( \xi_x \), is not very different from the high-frequency asymptotic value irrespective of basement shape and frequency. Thus, for practical applications, the approximation

\[
\xi_x \approx 1
\]

provides sufficient accuracy.

2. In the low frequency range, \( 0 \leq a_0 \leq 0.50 \), radiation damping during lateral oscillations is fairly sensitive to variations in foundation shape as well as frequency. Of all shapes considered, the strip (plane strain) produces the largest amount of damping per unit contact area, at all studied frequencies. By contrast, the circular and square shapes generate the lowest values of radiation damping per unit area. Rectangles exhibit an intermediate behavior, with \( \xi_y \) increasing (at each particular frequency) with increasing \( L/B \). These conclusions are somewhat paradoxical; waves under a circular footing may possibly spread into the half-space downward and outward in all directions, whereas under a strip, the geometric spreading of waves is restricted in two dimensions by the prevailing plane-strain conditions. On the basis of common sense, one might therefore have expected larger geometric (radiation) damping under axisymmetric conditions. The fact that in reality exactly the opposite holds true (Fig. 1, top) is a consequence of destructive interference, which limits severely the radial spreading of waves under a circular footing (2,5).

3. At high frequencies (\( a_0 > 1.50 \)) and for all foundation shapes, \( \xi_y \approx 1 \), confirming our earlier argument.

In conclusion, Fig. 1 is recommended for obtaining realistic estimates of the radiation damping coefficients of a surface foundation with an
FIG. 2.—Foundations in Open Trench: Effect of Depth and Aspect Ratios on Longitudinal and Lateral Radiation Dashpot Coefficients, $C_x$ and $C_y$

arbitrary (solid) shape having area $A_b$ and aspect ratio of the circumscribed rectangle $L/B$.

**FOUNDATION IN AN OPEN TRENCH ($d = 0$)**

The changes occurring in the damping coefficients $C_x$ and $C_y$ when the foundation is placed at the bottom of an open trench, without sidewall, can be assessed with the help of the seven charts shown in Fig. 2. Each chart corresponds to a single value of the aspect ratio, $L/B = 1, 4,$ and $6$. The different numerical points in each chart correspond to different values of the trench depth ratio, $D/B$, including $0.5, 1, 1.5,$ and $2$.

Evidently, the trench effect is rather insignificant for radiation damping (it is of the same order as the effect of Poisson’s ratio) and may thus be neglected. The interpretation of such behavior is straightforward; waves emanating from the basemat-soil interface spread basically downward, and thereby are hardly influenced by the mere presence of the overburden soil, as is the case with an open trench.

**EMBEDDED FOUNDATION—RADIATION FROM SIDEWALL AND BASE**

**Simple Physical Model.**—The radiation damping coefficients $C_x$ and $C_y$ represent the vibrational energy transmitted into the soil and carried away by outward and downward spreading waves. These are generated at every point on the soil-foundation interface (Huygen’s Principle) so that, in general, the damping coefficients increase with increasing area of contact. For swaying embedded foundations, in addition to shearing waves originating at the basemat, shearing and compression-extension waves are emitted from the vertical sides, depending on whether a particular side is parallel or perpendicular to the direction of loading. Inclined sides emit both types of waves.

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FIG. 3.—Embedded Foundation: Basic Sketch of Proposed Radiation Damping Model

The shearing waves emanating from the sides that are parallel to the direction of motion of the foundation propagate predominantly in the horizontal direction with an apparent velocity equal to the S-wave velocity, \( V_s \), of the surrounding soil. It is assumed that the travel of these waves is restricted in the horizontal direction and, thus, that they are not affected by the presence of the soil below the base. Therefore, the contribution to radiation damping, \( dC_{ws} \), from the shear waves originating at an element of the sidewall-soil interface of area \( dA_w \) and inclination \( \theta \) (Fig. 3) can be expressed as

\[
dC_{ws} = (\rho V_s dA_w \sin \theta) \hat{c}_s
\]

(5)

in which

\[
\hat{c}_s \approx \hat{c}_t \left( \frac{d}{a_0} \frac{D}{B'} \frac{L}{B} \right)
\]

(6a)

In fact, all the numerical evidence suggests that, as a first approximation, for \( D/B < 2 \)

\[
\hat{c}_s \approx 1
\]

(6b)

If this is the case, Eq. 5 represents one-dimensional damping due to S-waves propagating only in a direction perpendicular to the emitting surface.

On the other hand, the apparent propagation velocity \( V_\infty \) of the compression-extension wave energy is not the \( P \)-wave velocity, \( V_p \), as intuition might suggest and some researchers have tentatively assumed in the past. The reason is that \( P \)-waves travel without lateral restraining of the soil, while the group of compression-extension waves originating at the sidewalls does induce lateral deformations, especially by the free surface. After all, accepting \( V_p \) as the apparent wave velocity would lead one to expect the damping value to tend to infinity when Poisson’s ratio approaches 0.50 (i.e., in saturated soft clays). Neither such jump of damping values to infinity nor, in fact, any significant sensitivity of damping to Poisson’s ratio have been found in rigorous studies of these problems. Therefore, \( V_p \) is clearly inappropriate.

In Refs. 4 and 8, \( V_\infty \) was taken as equal to Lysmer’s analog velocity, \( V_{la} = 3.4V_s/[(1 - \nu)] \), for simulating the compression-extension wave energy radiated from the base of a vertically oscillating foundation and from the shaft of a laterally vibrating deep pile element. In the present case, it appears that the appropriate value for \( V_\infty \) would be primarily a
function of the degree of embedment, \( d/D \) (Fig. 3); as the latter is decreased, so is the proximity of the free surface and its influence on the group of compression-extension waves. Consequently, \( V_{ce} \) is expected to increase with decreasing \( d/D \). For fully embedded sidewalls (\( d/D = 1 \)), use of \( V_{ce} = V_{wa} \) yields results in agreement with the numerical solution, as shown in the sequel. Thus, in general, \( V_{ce} \geq V_{la} \).

The contribution to radiation damping, \( dC_{wa}\), of the compression-extension waves emanating from an element of the sidewall-soil interface of area \( dA_w \) and inclination \( \theta \) (Fig. 3) can be expressed as

\[
dC_{wa} = (\rho V_{ce} dA_w \cos \theta) \tilde{c}_{ce} \tag{7}
\]

in which

\[
V_{ce} \approx V_{la} = \frac{3.4V_s}{\pi(1 - v)} \tag{8}
\]

and

\[
\tilde{c}_{ce} = \tilde{c}_{ce} \left( d_0 ; \frac{d}{D}, \frac{D'}{B'}, \frac{L}{B} \right) \tag{9a}
\]

Again, the existing evidence suggests that, as a first approximation, for \( D/B < 2 \)

\[
\tilde{c}_{ce} = 1 \tag{9b}
\]

Integrating each of Eqs. 5 and 7 over the whole area of sidewall-soil contact surface and summing up the respective radiated energies leads to the following expression for the contribution \( C_w \) of the sidewall to the radiation dashpot coefficient:

\[
C_w = C_{wa} + C_{wa} = \left( \rho V_s A_{ws} \right) \tilde{c}_e + \left( \rho V_{ce} A_{wa} \right) \tilde{c}_{ce} \approx \rho V_s A_{ws} + \rho V_{ce} A_{wa} \tag{10}
\]

in which:

\[
A_{ws} = \sum_i \left( A_{wi} \sin \theta_i \right) \tag{11}
\]

is the sum of the projections of all the sidewall areas in a direction parallel to loading, i.e., the effective area of the sidewall surface shearing the soil, and

\[
A_{wa} = \sum_i \left( A_{wi} \cos \theta_i \right) \tag{12}
\]

is the sum of the projections of all the sidewall areas in a direction perpendicular to the loading, i.e., the effective area of the sidewall surface compressing the soil. Eq. 10 is applicable for both \( x \) and \( y \) directions, with \( C_w = C_{wx} \) and \( C_w = C_{wy} \), respectively.

Finally, assuming that basemat and sidewalls radiate independently of each other and summing up the respective energies leads to the total radiation dashpot coefficient:

\[
C \approx C_b + C_w = \left( \rho V_s A_b \right) \tilde{c} + \rho V_s A_{ws} + \rho V_{ce} A_{wa} \tag{13}
\]

in which, depending on the loading direction \( C = C_x \) or \( C_y \), \( C_b = C_{bx} \) or \( C_{by} \) (Eq. 3a); and \( \tilde{c} = \tilde{c}_x \) or \( \tilde{c}_y \) (Eq. 3b and Fig. 1). The general validity of Eqs. 10 and 13 is explored in the sequel.

Numerical Data: Radiation from the Sidewall.—This section investigates the ability of the simple model (Eqs. 10–12) to provide realistic estimates of the radiation dashpot coefficients \( C_w \) (\( C_{wx} \) or \( C_{wy} \)) contrib-
FIG. 4.—Radiation from Cylindrical Sidewall-Soil Interface: Numerical Solution Showing Effects of Complete and Partial Nonsymmetric Contact ($\nu = 0.40$)

FIG. 5.—Radiation from Sidewalls of Triangular Foundation: Numerical Solution Showing Effects of Complete and Partial (Nonsymmetric) Contact ($\nu = 0.40$)

ured by the sidewalls of an arbitrarily-shaped foundation. To this end, foundations with perimeter of circular, triangular, and rectangular shapes are studied, each having several types of symmetric and nonsymmetric contact (Figs. 4–6). To isolate the effects of the sidewalls from those of the basemat, the soil surrounding the rigid foundation on the sides is treated as a plane laterally loaded elastic disk in a way similar to that introduced by Novak and his coworkers (16,17) for embedded circular foundations and piles.

For the circular foundation (Fig. 4), the cylindrical wall is assumed either in complete (case 1) or in partial (cases 2–5) contact with the surrounding soil, as in the work by Chen (1) on nonuniformly embedded foundations. Cases 2–5 have identical contact areas, covering only 1/2
FIG. 6.—Radiation from Sidewalls of $L/B = 2$—Rectangular Foundation: Numerical Solution for Complete and Partial Contact ($\nu = 0.40$)

of the foundation perimeter, but differ with respect to the extent of the soil mass in the vicinity of the unattached side.

For case 1 (complete contact), both for $x$ and $y$ directions, Eqs. 11–12 predict $A_{w_1} = A_{w_2} = 4RD$ where $R =$ the foundation radius; and $D =$ the thickness of the soil disk. For all four cases 2–5 (partial nonsymmetric contact), and, again, both for $x$ and $y$, Eqs. 11–12 predict $A_{w_1} = A_{w_2} = 2RD$. Hence, Eq. 10 suggests that $C_{w_2} = C_{w_3} = C_{w_4} = C_{w_5} = (1/2)C_{w_1}$, in both directions. Indeed, the numerical (boundary-element) solution plotted in Fig. 4 overwhelmingly confirms the fact that, regardless of frequency and geometric details, the dashpot coefficients are the same in all four partial-contact cases, equal to about 1/2 of the dashpot coefficient in the complete contact case.

The basic validity of the simple model (Eqs. 10–12) is also amply verified by the results for an equilateral triangle and an $L/B = 2$ rectangular foundation plan shapes (Fig. 5 and 6). For the triangular foundation, three types of interface are studied, with all three (case 3) or only two (case 2) or only one (case 1) of the sidewalls assumed in contact with the surrounding soil. Several predictions of the model can be readily verified with the help of Fig. 5. For example, it is easily checked in this figure that $C_{w_1} + C_{w_2} \approx C_{w_3}$ in both $x$ and $y$ directions, in accord with Eqs. 10–12. Another example, for case 1 and motion in the $y$ direction, $A_{w_1} = 2BD$ and $A_{w_2} = 0$ (Eqs. 11–12); thus, from Eq. 10, $C_w = 2pV_s BD$ and, therefore, $\omega C_w/GD = 2a_w$ which is in very good agreement with the corresponding curve (essentially straight line) plotted in Fig. 5.

The predictions of the simple model compare favorably with the numerical results for the rectangular foundation (Fig. 6) as well. For example, it may be readily checked that, again, for both directions, $C_{w_1} + C_{w_2} \approx C_{w_3}$, where cases 1 and 2 refer to sidewall-soil contact only along two (opposite) sides of the rectangle, while case 3 refers to complete contact along all four sides.

In conclusion, the numerical data of Figs. 4–6 provide an adequate
corroboration of the basic assumptions of Eqs. 10–12: various parts of the sidewall-soil interface radiate independently of one another and each part radiates both shear and compression-extension waves that carry amounts of energy in proportion to the areas of the projections on planes parallel and perpendicular to the direction of loading, respectively.

Numerical Data: Fully Embedded Foundations.—This section studies foundations embedded at depth $D$ having sidewalls of height $d = D$, in perfect contact with the surrounding soil.

The combined effect of the depth ratio, $D/B$, and the aspect ratio, $L/B$, on the normalized damping coefficients $C_s/pV_s A_s$ and $C_y/pV_s A_s$ is shown in Fig. 7. It is evident that while increasing $D/B$ would invariably increase the radiation damping, the magnitude of the increase is a function of both $L/B$ and the direction of oscillation. It is of interest to explore the ability of Eq. 13 to anticipate the trends observed in Fig. 7 and, perhaps, to provide results in quantitative agreement with the rigorous numerical solution.

As mentioned earlier, for fully embedded sidewalls ($d = D$), the choice $V_{ce} = V_{La}$ leads to results that are consistent with the numerical data. Hence, $V_{La}$ is used throughout this section.

For a rectangular foundation with plan dimension $2B$ by $2L$ ($L \geq B$) fully embedded at depth $D$ in a halfspace with $v = 0.40$, Eq. 13 gives:

(a) for motion in the lateral, $y$, direction:

$$\frac{C_y}{pV_s A_s} \approx \tilde{c}_y (a_0) + \frac{D}{B} \left[ 1.80 + \frac{1}{L/B} \right]$$

(14a)

FIG. 7.—Fully Embedded Rectangular Foundations: Effect of Depth and Aspect Ratios on Longitudinal and Lateral Radiation Dashpot Coefficients, $C_s$ and $C_y$ (Data for Strip: Ref. 18)
for motion in the longitudinal, \( x \), direction:

\[
\frac{C_x}{\rho V_x A_b} \approx \bar{c}_x(a_0) + \frac{D}{B} \left[ 1 + \frac{1.80}{L/B} \right]
\]

(14b)

The aforementioned trends observed in Fig. 7 can be readily explained with the help of the above two expressions. Thus, for a given \( D/B \), increasing the aspect ratio \( L/B \) would tend to reduce the damping coefficient per unit area of the foundation plan. For instance, the depth-dependent factor in Eq. 14a decreases from \( 2.80D/B \) for a square plan \( (L/B = 1) \) to \( 1.80D/B \) for a strip plan \( (L/B = \infty) \). The effect of the direction \( (x \) versus \( y \)) is also evident by comparing the above two expressions. As an example, for \( L/B = 4 \), the depth-dependent factor equals \( 2.05D/B \) for \( C_y \), but is only equal to \( 1.45D/B \) for \( C_x \).

A quantitative comparison between the predictions of the simple model and the results of rigorous numerical analyses shown in Fig. 8 (for squares and circles) and Fig. 9 (for rectangles with \( L/B = 4 \) and 10). The lines in these figures are plots of Eq. 13 (or Eq. 14). The numerical data points are from the writers' boundary-element solution, except those for the cylindrical foundation which come from Day (3).

The performance of the simple radiation damping model (Eqs. 13–14) is very good in all cases studied (not only those shown in Figs. 8 and 9) and for all frequencies. The maximum error in the frequency range considered does not exceed 20%, and in most cases it stays less than merely 5%. It is worth keeping in mind that even results from different

\[\text{FIG. 8.—Comparison of Proposed Model Predictions (Eq. 13) with Rigorous Results for Damping of Square and Circular Embedded Foundations}\]
rigorous solutions may show discrepancies of the order of 10%; such discrepancies arise from different assumptions regarding the boundary conditions imposed by the foundation basemat and sidewalls (smooth versus rough interface), different solution techniques, and different discretization accuracies. Thus, the proposed simple model is recommended for practical applications as long as $D/B < 2$. For greater embedment depths, however, the model does not seem capable of reproducing the observed trend of $C_y$ increasing rapidly with frequency, for $a_0 > 1$. The numerical data in the graphs of Figs. 7–9 can nevertheless be used as a guide for such large depths and frequencies.

**Numerical Data: Partial Sidewall Contact.**—This section studies foundations embedded at depth $D$ but having sidewalls which are in contact with the surrounding soil only over a height $d < D$.

The effect of the ratio $d/D$ of the sidewall–soil contact height to trench depth is shown in Fig. 10. This figure encompasses five charts portraying $C_z/pV_s A_b$ and $C_y/pV_s A_b$ versus $a_0$ and $d/D$, for foundations with aspect ratios $L/B = 1, 4$, and $10$, and a relative trench depth $D/B = 1$. Three or four values of $d/D$ are considered, ranging from 0 (foundation in an open trench) to 1 (fully embedded foundation).
A consistent trend is evident in this figure: while the area $A_w$ of the sidewall-soil interface increases linearly with $d/D$, the damping coefficients exhibit a strongly nonlinear increase; for small values of $d/D$, say less than 0.50, damping increases at a faster rate than $A_w$, whereas for larger values of $d/D$ damping increases slower than $A_w$. Thus, when $d/D = 2/3$, the damping coefficients attain only slightly lower values than those of the fully embedded foundation.

An explanation of this trend is affordable within the framework of the simple model (Eq. 13) presented in this paper. At small values of $d/D$, the compression-extension waves emitted from the sidewalls are hardly influenced by the free surface; thus, their apparent propagation velocity, $V_{ae}$, should be larger than the apparent velocity, $V_{ae}$, which was found appropriate for sidewalls extending all the way to the surface ($d = D$). As $d/D$ grows larger, the free surface gets closer to the oscillating wall and the apparent velocity decreases.

As a conclusion, it is hardly efficient to try to improve the radiation damping characteristics of an embedded foundation by increasing the height of its sidewalls beyond about 2/3 of the depth of embedment. Conversely, the designer of a swaying foundation does not have much cause to worry about separation and slippage between the walls and the
backfill near the ground surface; even if such phenomena do occur, they will probably have only a minor adverse effect on radiation damping in swaying. However, a note of caution is appropriate: rocking stiffness and damping may be highly sensitive to such near-surface phenomena.

FURTHER COMPARISONS WITH RIGOROUS RESULTS

As discussed in detail in the companion paper (10), only a limited number of parametric results from rigorous solutions have been published so far for foundations embedded in a halfspace, in Refs. 3 and 20 for a circular base, in Ref. 20 for an infinitely long foundation under plane conditions, and in Ref. 6 for a rectangular base of aspect ratio \( L/B = 1 \) and 2. More extensive results have been published for surface circular, strip, and rectangular foundations on a halfspace (e.g., Refs. 9, 12, 13, 19, and 21), and for cylindrical foundations embedded in a shallow stratum over a rigid base-rock (e.g., Refs. 11, 20). On the other hand, the effects of embedment on the response of foundations in deep soil deposits is usually being assessed using the versatile approximate method of Novak (16,17).

In this paper, the charts for \( \tilde{c}_y \) and \( \tilde{c}_x \) for surface foundations (Fig. 1) have been based on data from Refs. 5, 9, 12, 13, 19, and 21, in addition to our own boundary-element results. The results of Day (3) for cylindrical embedded foundations have been compared with the predictions of the presented method in Fig. 8. The results of Dominguez (6) for rectangular embedded foundations with \( L/B = 1 \) and 2 compare very well with our results; after all, the boundary-element formulation utilized herein is a refined version of Dominguez's technique. Also very good is the agreement with the results of Wolf (21) for an embedded strip.

Given in Fig. 11 is further demonstration of the capability of Eq. 13 to predict with good accuracy the radiation damping of arbitrarily shaped foundations. The figure refers to two foundations having T-shaped basemats and placed either on the surface, or embedded at depth \( D = B \). The circumscribed rectangles have \( L/B = 1 \) and 2, respectively. The rigorous results are from the aforementioned boundary-element formulation.

The geometry of the first basemat gives

\[
A_b = \frac{8}{3} B^2, \quad A_{\omega_b} = Q_{\omega_{bx}} = 4BD
\]

for both \( x \) and \( y \) directions. For \( v = 0.40 \), Eq. 13 yields

\[
\frac{C}{\rho V_s A_b} = \tilde{c} \left( \frac{L}{B} = 1 \right) + 4.2 \frac{D}{B}
\]

independently of loading direction. The values of \( \tilde{c} \) for \( L/B = 1 \) are read from Fig. 1.

The performance of Eq. 16, plotted in Fig. 11, is excellent for the surface foundation \( (D/B = 0) \) and very satisfactory (error less than 20%) for the embedded foundation \( (D/B = 1) \). Indeed, damping is practically independent of the direction \( (x \text{ or } y) \) of oscillation.
FIG. 11.—Evaluation of Proposed Method (Eq. 13) by Comparing its Results for T-Shaped Surface and Embedded Foundations with Rigorous Numerical Solutions

For the second basement, and the \( x \) loading direction

\[ A_b = 5.33B^2, A_{w_x} = 8BD, A_{w_y} = 4BD \]  \hspace{2cm} (17)

and thus

\[ \frac{C_x}{\rho V_x A_b} = \tilde{c}_x \left( \frac{L}{B} = 2 \right) + 2.85 \frac{D}{B} \]  \hspace{2cm} (18)

while in the \( y \) direction

\[ A_b = 5.33B^2, A_{w_x} = 4BD, A_{w_y} = 8BD \]  \hspace{2cm} (19)

leading to

\[ \frac{C_y}{\rho V_y A_b} = \tilde{c}_y \left( \frac{L}{B} = 2 \right) + 3.46 \frac{D}{B} \]  \hspace{2cm} (20)

where \( \tilde{c}_x \) and \( \tilde{c}_y \) for \( L/B = 2 \) are read for each frequency from Fig. 1.

The performance of Eqs. 18 and 20 is very good (error less than 15%) for both surface and embedded foundations. Observe that, despite its simplicity, the method developed in this work is capable of anticipating even subtle trends in the response, such as the difference between \( C_x \) and \( C_y \) of the second foundation.

**Two Illustrative Examples**

The formulas and the charts presented above are utilized in this section to obtain estimates of the damping coefficients in two examples dealing with the same embedded foundations studied in the companion paper (9).
Example I.—Of interest is $C_y$ for the hypothetical embedded foundation shown in Fig. 12. The basemat is a truncated rectangle, placed 8 m below the ground surface. A 6-m high vertical sidewall, in perfect contact with the surrounding soil, is built around the periphery. The excitation frequency $\omega$ is equal to $35 \text{ rad/sec}$.

The foundation basemat has an area $A_b = 617.5 \text{ m}^2$, and aspect ratio of circumscribed rectangle $L/B = 3$. The sidewall height, $d$, is uniform along the perimeter, equal to $3/4$ of the trench depth, $D$. From the preceding discussion on the effect of $d/D$ (Fig. 11), it is concluded that the damping coefficient $C_y$ in this case may be estimated using Eq. 13 with $V_{ce} = V_{la}$ and assuming that the sidewall extends all the way to the surface ($d = D = 8$ m). For $v = 0.35$ and $V_s = 194 \text{ m/sec}$: $V_{la} \approx 323 \text{ m/sec}$.

The effective sidewall areas: $A_{ave} = 30 \times 8 = 240 \text{ m}^2$ and $A_{ave} = 90 \times 8 = 720 \text{ m}^2$. For the frequency factor $a_0 = 1.35$ and $L/B = 3$, we read (by interpolation) in Fig. 2: $\zeta_y \approx 1$. Thus, Eq. 13 gives

$$C_y \approx (1.8 \times 194 \times 617.5) \times 1 + (1.8 \times 194 \times 240) + (1.8 \times 323 \times 720)$$

$$= 2.16 \times 10^5 + 0.83 \times 10^5 + 4.19 \times 10^5 \approx 7.2 \times 10^5 \text{ Mg/sec} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (21)$$

(Note that $1 \text{ Mg/sec} = 1 \text{ Mg ms}^{-2}/\text{ms}^{-2}\text{sec} = 1 \text{kN m}^{-1} \text{sec}$.)

Example II.—The basemat of the foundation sketched in Fig. 13 is composed of a rectangle, a trapezium, and a semi-circle. Part of its perimeter is connected to a 7-m high vertical sidewall, assumed to be in perfect contact with the backfill. No sidewall (and, thus, no contact) exists along the side $abcd$. The geometric and material parameters of the system are listed in Fig. 13. Of interest is the horizontal dashpot coefficient $C_y$ for an excitation frequency $\omega = 40 \text{ rad/sec}$.

We first notice that the height of the existing sidewall is equal to the depth of the trench, $d = D = 7$ m. The compression-extension waves from such a wall would propagate with velocity $V_{la} = 3.4 \times 170/\pi(1.0 - 0.40) = 307 \text{ m/sec}$. The existence of an open trench on one side of the foundation presents no special computational problem. For oscillation in the $x$ direction, the effective sidewall areas are: $A_{ave} = 32 \times 7 = 224 \text{ m}^2$ and $A_{ave} = 10 \times 7 = 70 \text{ m}^2$. Recall that $A_b = 170 \text{ m}^2$. For $a_0 = 1.18$ and $L/B = 1.9$, Fig. 2 yields $\zeta_s \approx 0.92$. Thus, from Eq. 13:

![Diagram](attachment:image.png)

**FIG. 12.—Illustrative Example I: Geometry and Material Parameters**
FIG. 13.—Illustrative Example II: Geometry and Material Parameters

\[ C_r = (1.75 \times 170 \times 170) \times 0.92 + (1.75 \times 170 \times 224) + (1.75 \times 307 \times 70) = 0.47 \times 10^5 + 0.67 \times 10^5 + 0.38 \times 10^5 \approx 1.5 \times 10^5 \text{ Mg/sec} \]  

(22)

**Extension of Model to Inhomogeneous Profiles**

Although the radiation damping model described in this paper has been developed and calibrated with the help of numerical data for a homogeneous halfspace, its basic framework can encompass more realistic soil profiles. As an example, such a profile is shown in Fig. 14(a); it consists of a backfill soil layer with modulus \( G_1 \) underlain by an inhomogeneous halfspace with modulus \( G_2(1 + z/B)^{1/2} \), where \( z \) is the depth below the base.

One of the key assumptions of the model, namely, that the sidewall and the basemat radiate independently of each other, would still be approximately valid, despite the different stiffnesses of the backfill and the subsoil (16). Moreover, the inhomogeneity of the subsoil would change only the dimensionless frequency-dependent coefficients \( \tilde{e}_x \) and \( \tilde{e}_y \) in Eq. 3. The new coefficients, \( \tilde{e}_x^* \) and \( \tilde{e}_y^* \), can be found from the response of a surface founation on the same soil profile (e.g., Ref. 8); for this particular case of modulus increasing parabolically with depth, they are plotted in Fig. 14(b). Thus, for a fully embedded foundation \( d = D \), the radiation damping can be estimated from the expression (valid for either direction)

\[ C = (\rho_2 V_{s2} A_b) \tilde{e}_x^* + \rho_1 V_{s1} A_{ax} + \rho_1 V_{L1} A_{sx} \]  

(24)

in which: \( \rho_1, V_{s1}, \text{ and } V_{L1} = \text{mass density, } S\text{-wave velocity, and } \text{Lysmer's analog velocity of the backfill; while } \rho_2 = \text{mass density of the sub-} \)
soil; and $V_{s2} = S$-wave velocity of the subsoil at $z = 0$, i.e., directly underneath the base. Partial contact ($d < D$) can be treated (by interpolation) with the help of the charts of Fig. 10.

**Conclusion**

Simple algebraic formulas and dimensionless parametric graphs have been presented for estimating the radiation dashpot coefficients, $C_x$ and $C_y$, of horizontally oscillating rigid foundations that are embedded in a halfspace and have arbitrary base shapes and partial or complete (symmetric or nonsymmetric) sidewall-soil contact. Two numerical examples, involving odd geometries, have been given in order to illustrate the estimation procedure. Although the rigorous boundary-element results utilized in this study refer only to a homogeneous halfspace, the conceptual framework (model) developed in the paper has been readily extended to a more general and realistic inhomogeneous soil profile. Extension of the model to approximately treat the presence of bedrock at a shallow depth below the foundation base seems equally feasible but is not discussed in this paper.

**Appendix.—References**


