Seismic response of end-bearing single piles

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A numerical study is presented of the dynamic response of end-bearing piles embedded in a number of idealized soil deposits and subjected to vertically propagating harmonic S-waves. Results, for both 'kinematic' and 'inertial' interaction, are offered in the form of dimensionless graphs and formulae covering a wide range of excitation frequencies and of crucial material and geometric parameters. Practical aspects of the evaluation of the influence of piles on the effective seismic excitation of a structure are discussed and a case history illustrates the usefulness of the presented results.

Key Words: dynamic response, end-bearing piles, soil deposits, harmonic S-waves, seismic excitation.

INTRODUCTION

The response of piles to vertically propagating S-waves has been studied by several authors.¹⁻¹² Despite the significant progress in understanding the seismic behavior of single piles, several questions remain unanswered, especially with regard to the practical assessment of the influence of piles on the seismic excitation of a structure. For instance, it has been suggested that, since flexible piles follow the ground motion, there is no need to modify the input excitation. Consequently, analysis procedures often make use of the same design response spectra as for structures on shallow foundations.^{3, 13, 14} The validity of this approximation, however, has not yet been adequately verified. In fact, the results of this study show that, in certain cases, even relatively long piles may appreciably modify the base excitation of a supported structure.

Furthermore, only a very limited number of results are available in the form of dimensionless graphs and formulae. Such results would be useful not only for developing an improved understanding of the mechanics of the problem and checking the accuracy of sophisticated solutions, but also for making preliminary design estimates in practice. By contrast, several parametric studies have been published for piles subjected at their head to horizontal static^{4, 15-20} or dynamic^{4, 7, 11, 19-25} forces.

This paper presents an extensive parameter study, conducted with the efficient finite-element formulation developed by Roesset and his co-workers.4, 24 Results are presented in the form of dimensionless soil-pile interaction and amplification factors, as well as pile-head impedance functions. Among the groups of problem parameters influencing the response most important (for each particular soil profile) have been found to be: the stiffness ratio E_p/E_s of the pile Young's modulus over a characteristic Young's modulus of the soil deposit; the slenderness ratio L/d of the length over the diameter of the pile; the frequency ratio f/f_1 of the excitation frequency over the fundamental natural frequency of the unperturbed soil deposit in vertical S-waves; and the relative frequency factor f_{st}/f_1 , where f_{st} is the fundamental frequency of the pile-supported superstructure. Conclusions are drawn on the validity of current seismic design practices and a case history is presented involving actual earthquake records on, and nearby, a piled foundation. The usefulness of the results offered in the paper is illustrated by comparing theoretical and recorded transfer functions.

STATEMENT OF THE PROBLEM

The system studied refers to an end-bearing pile supporting a block of mass M (super-structure) and being embedded in a soil stratum of thickness L (Fig. 1a). Vertically incident harmonic S-waves constitute the base excitation, which is described through the displacement: $u_g(t) = u_g \cdot \exp(i2\pi f t)$. The pile is a linearly elastic flexural beam with a circular cross-section of diameter d, Young's modulus E_p and mass density ρ_p . The soil is modeled as a linearly hysteretic continuum with constant Poisson's ratio v_s , mass density ρ_s and hysteretic damping ratio β_s , but with Young's modulus E(z) which varies with depth from the ground surface.

Three soil models are considered, each with a different variation of E(z), as sketched in Fig. 2. In Model A, E(z) is proportional to depth, representing uniform soft normallyconsolidated clay deposits. In Model B, E(z) is proportional to the square root of z – an idealization appropriate for uniform deposits of cohesionless soils. Finally, Model C has a modulus $E(z) = E_s$, constant with depth – typical of stiff overconsolidated clay deposits. These three models may adequately represent the dynamic characteristics of a fairly wide range of soil profiles encountered in nature.

It is conceptually attractive and computationally convenient to express the response of the system shown in Fig. 1a as a superposition of two effects: ${}^{3, 14, 26, 27}$ (1) a kinematic interaction effect, involving the response to baserock excitation of the system shown in Fig. 1b, which differs from the complete system of Fig. 1a in that the mass of the super-structure is set equal to zero; (2) an inertial interaction effect, referring to the response of the complete pile-soil-structure system to excitation by D'Alembert forces, $-M\ddot{u}_k$, associated with the acceleration, \ddot{u}_k , of the super-structure due to the kinematic interaction (Fig. 1c). This superposition is exact if the analyses in both steps are rigorously performed. The popularity, however, of the approach stems from a suggested approximation to the

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kinematic interaction effects:^{3, 14, 26} since in many cases piles tend to follow the ground, soil-pile interaction is ignored and the free-field motion is used as input in the inertial interaction step. One of the purposes of this paper is to comment on the range of validity of this approximation.

Another potentially significant simplification may also be suggested in the context of the kinematic-inertial decomposition.¹⁴ First, recall^{16, 18, 19, 22} that, in the majority of actual cases, pile deformations due to lateral excitation



Figure 1. (a) Geometry of soil-pile-structure interaction problem; (b) decomposition into kinematic and inertial interaction problems; (c) two-step analysis of inertial interaction

transmitted from the super-structure attenuate very rapidly with depth (typically within 10-15 diameters from the ground surface). Therefore, shear strains induced in the soil due to inertial interaction may be significant only near the ground surface.^{6, 28} By contrast, vertical S-waves induce in the free-field shear strains that are likely to be important only at relatively deep elevations. Thus, since soil strains are controlled by inertial effects near the ground surface and by kinematic effects at greater depths, the superposition may be approximately valid even if nonlinear soil behavior is expected, during a strong base excitation.

Note also that, as originally proposed by Kausel and Roesset²⁷ for embedded foundations, inertial interaction analyses in the frequency domain can be conveniently performed in two steps, sketched in Fig. 1c. Determination of the dynamic impedances, \mathscr{K}_{HH} , \mathscr{K}_{MM} and \mathscr{K}_{HM} , which express dynamic force-displacement ratios at the head of the pile, is a central task of this approach.

Finally, kinematic-inertial decomposition is particularly suitable for parametric studies, and provides considerable insight into the mechanics of pile-soil-structure interaction.

PARAMETRIC RESULTS: KINEMATIC INTERACTION

In the absence of a pile, a vertically incident S-wave would induce only horizontal displacements in the free-field soil. For a baserock motion $u_g(t) = u_g \exp(i2\pi ft)$ and a homogeneous soil stratum, the one-dimensional 'amplification' theory²⁹ would give for the steady-state free-field displacement at ground surface level:

$$u_0(t) = u_0 \exp(i2\pi f t) \tag{1a}$$

$$\frac{u_0}{u_g} = \frac{2}{\exp(iqL) + \exp(-iqL)}$$
(1b)

(*L* is the stratum thickness and $q^2 = 4\pi^2 f^2 / [V_s^2(1+2i\beta_s)]$, where V_s and β_s are the S-wave velocity and the internal (hysteretic) damping in the soil; $i^2 = -1$.) If $\beta_s = 0$, equation (1b) yields $u_0/u_g = 1/\cos(2\pi fL/V_s)$ which tends to infinity (resonance) at the natural shear frequencies of the stratum, $f_n = (2n-1) V_s/4L$, n = 1, 2, 3, ...

A cylindrical pile diffracts the incident and reflected one-dimensional vertical S-waves, thereby modifying the 'free' wave field. As a result, the horizontal displacement atop the pile, $u_p(t) = u_p \exp(i2\pi ft)$ differs from $u_0(t)$ of equation (1). In addition, the pile top experiences a rotation, $\phi_p(t) = \phi_p \exp(i2\pi ft)$. It is convenient to portray the effects of kinematic interaction by introducing the displacement and rotation kinematic interaction factors

$$I_u = \frac{u_p}{u_0} \text{ and } I_\phi = \frac{\phi_p r_0}{u_0}$$
 (2)



Figure 2. The three soil models studied

and also, the displacement and rotation kinematic amplification factors

$$A_{\mu} = \frac{u_p}{u_g} \quad \text{and} \quad A_{\phi} = \frac{\phi_p r_0}{u_g} \tag{3}$$

where $r_0 = d/2$ is the radius of the pile, while u_p and u_0 are the amplitudes of horizontal displacement of the pile top and the free-field ground surface, respectively, relative to the base; the total respective displacements are $u_p(t) + u_g(t)$ and $u_0(t) + u_g(t)$. Without kinematic interaction u_p would be equal to u_0 , and ϕ_p equal to zero. Then, $I_u = 1$, $I_{\phi} = A_{\phi} = 0$ and A_u would be given by equation (1b). In the sequel, kinematic interaction effects are studied in terms of these four interaction and amplification factors.

Because of the presence of both radiation (due to diffraction) and material damping in the system, the various displacement and rotation components are not in-phase with the excitation. It has been customary in the soil dynamics literature to use complex notation in order to express differences in phase. In this sense, the four amplification and interaction factors are complex functions of frequency. Only their absolute values (amplitudes) are studied herein; this is usually sufficient for practical applications.¹⁸

Figures 3-5 present parametric results for the variation of kinematic amplification and interaction factors versus

 Table 1. Expressions for natural shear frequencies of the three soil deposits

Soil model	f_i	f_2/f_1	
A	$1.21 V_{s}/H$	2.33	
В	$0.56 V_s/H$	2.66	
С	$0.25 V_{s}/H$	3.00	

 $V_s = S$ -wave velocity at depth z = d below the ground surface

the frequency ratio f/f_1 . f_1 is the fundamental shear frequency (in Hz) of each stratum, computed from the expressions of Table 1 for each of the three soil profiles in Fig. 2.³⁰

Effect of stiffness ratio E_p/E_s

The influence of E_p/E_s is portrayed in Fig. 3 for a pile having L/d = 40, $\rho_p/\rho_s = 1.60$ (typical of concrete piles) and being embedded in soil Model A with $\nu_s = 0.40$ and $\beta_s = 0.05$. The following trends are worthy of note in Fig 3.

1. The presence of the pile and the value of E_p/E_s do not have any influence on the first resonant frequency, which practically coincides with f_1 , the fundamental shear frequency of the unperturbed soil stratum. Therefore, in the sequel, f_1 is used to denote both frequencies, indiscriminately.



Figure 3. Kinematic interaction effects: influence of E_p/E_s (L/d = 40; soil Model A; $\beta_s = 0.05$, $\nu_s = 0.40$, $\rho_p/\rho_s = 1.60$)

- 2. Up to a frequency of about $1.50 f_1$, piles of all relative stiffnesses appear to essentially follow the movement of the ground; hence, their presence has no practical effect on the seismic motion at ground-surface level.
- 3. On the other hand, at higher frequencies even practically flexible piles $(E_p/E_s \text{ as low as 290})$ may not be able to follow the wavy movements of the free-field and may thereby experience considerably reduced deformations. This filtering effect is substantial for stiffer piles $(E_p/E_s > 20\,000)$, to the point that after the second natural frequency, the pile seems to remain essentially still, while the free-field soil mass moves considerably. This is in agreement with the actual earthquake observations of the response of a pile foundation, as reported by Otha *et al.*³¹ and Tajimi.⁹
- 4. As already mentioned, a rotational component of motion developed at the head of a pile in addition to the translational one (this component is not present in the free-field surface motion). The rotation exhibits several peaks at the natural shear frequencies of the deposit. Note that for relatively soft piles the peaks of rotation increase substantially at the higher natural frequencies; the opposite is true for relatively stiff piles, in accord with their smaller displacements.

The effect of E_p/E_s on the kinematic response of piles embedded in the other two soil models, B and C, is of a similar nature.

Effect of soil profile

Typical similarities and differences in kinematic interaction due to differences in the type of soil profile may be seen in Fig. 4. This figure compares the variation of the amplification and interaction factors versus f/f_1 , for a pile embedded in each of the three soil Models A, B and C. In all three cases: L/d = 40, $v_s = 0.40$ and $\beta_s = 0.05$. However, E_p/E_s is equal to 145 000 in the two inhomogeneous Models, A and B, and equal to 50 000 in the homogeneous Model C. This choice of moduli was made in order to compare pile-soil systems with similar overall stiffness. The three curves in each graph of Fig. 4 serve to qualitatively, rather than quantitatively, highlight the following trends associated with each model:

- 1. In all cases, the second resonance occurs at a frequency approximately equal to f_2 , the second natural shear frequency of the respective unperturbed stratum. Figure 4, in agreement with Table 1, shows that f_2 gets closer to f_1 as the degree of soil inhomogeneity increases (from C to A).
- 2. A substantial increase is observed in the peak rotation at $f = f_1$ as the degree of soil inhomogeneity increases. By contrast, the two resonant peaks of horizontal displacement as well as the second resonant peak of rotation are less sensitive to differences in the soil profile.
- 3. The horizontal interaction functions $I_u = I_u(f/f_1)$ reveal that, in the frequency range studied $(f/f_1 \le 4)$, the filtering by a pile of the high frequency components of the base excitation may be substantially greater with inhomogeneous than with homogeneous soil deposits. At the same time, the rotational components of motion may also be stronger in the inhomogeneous deposits.



Figure 4. Kinematic interaction effects: influence of the type of soil profile $(L/d = 40; \beta_s = 0.05, \nu_s = 0.40, \rho_p/\rho_s = 1.60)$



Figure 5. Kinematic interaction effects: influence of L/d ($E_p/E_s = 145\ 000$; soil Model B; $\beta_s = 0.05$, $\nu_s = 0.40$, $\rho_p/\rho_s = 1.60$)

Effect of slenderness ratio L/d

Figure 5 portrays the effect of L/d on the kinematic interaction/amplification for piles with $E_p/E_s = 145\,000$ in soil Model B (modulus proportional to v_z). Three values of L/d are examined, 10, 20 and 40, which cover a fairly wide range of slenderness ratios of actual piles.

It is evident that L/d has a profound effect on pile-head rotation, at all frequencies. On the other hand, its influence on displacements becomes appreciable only at frequencies greater than about 1.50 f_1 , when the shorter piles produce stronger filtering effects.

The rotation amplitudes at frequencies $f < f_1$ are understandably higher for the shorter piles which experience about the same relative displacement as the longer piles but over a shorter length. However, at $f \approx f_2$ short piles experience relatively small rotations, consistent with their small (due to filtering) displacements.

Synthesis of results for I_u

The value of I_u from all cases studied are replotted in Fig. 6, for each of the three soil profiles. Figure 6a applies to Profile A and portrays the variation of I_u versus the dimensionless frequency parameter

$$F_{A} = \frac{f}{f_{1}} \cdot \left(\frac{E_{p}}{E_{s}}\right)^{0.10} \cdot \left(\frac{L}{d}\right)^{-0.40} \tag{4}$$

for Profile B, Fig. 6b plots I_u as a function of

$$F_B = \frac{f}{f_1} \cdot \left(\frac{E_p}{E_s}\right)^{0.16} \cdot \left(\frac{L}{d}\right)^{-0.35}$$
(5)

Finally, for Profile C, Fig. 6c shows I_u as a function of

$$F_{C} = \frac{f}{f_{1}} \cdot \left(\frac{E_{p}}{E_{s}}\right)^{0.30} \cdot \left(\frac{L}{d}\right)^{-0.50}$$
(6)

Determined by trial and error so that the results fall within a relatively narrow band, each of these dimensionless parameters encompasses the three key normalized problem parameters, E_p/E_s , L/d and f/f_1 . The scatter of each set of 'data points' around the respective 'average' curve is very small for practical applications. Therefore, the value of the kinematic displacement interaction factor, I_u , may be estimated readily and with engineering accuracy from the 'average' curve of the pertinent soil model.

It is noted that the 'average' I_{μ} curves of Fig. 6 are of a similar nature to those proposed by Elsabee *et al.*³² for



Figure 6. Kinematic interaction factors, I_u , in terms of the dimensionless frequency parameters F_A , F_B and F_C (equations (4)-(6))

embedded circular foundations. The practical significance of such curves is apparent: by multiplying a given free-field design response spectrum with the appropriate interaction curve, one may derive the design response spectrum that must be input at the base of a structure on piles foundation. At the same time, however, the rotational component of the input motion, expressed through I_{ϕ} , should not be neglected.

PARAMETRIC RESULTS: DYNAMIC IMPEDANCES

The steady-state response of piles to lateral dynamic forces transmitted from the super-structure can be easily computed once the three dynamic impedances, \mathscr{K}_{HH} , \mathscr{K}_{MM} and \mathscr{K}_{HM} , associated with swaying, rocking and coupled swaying-rocking oscillations, respectively, have been derived. These impedances are ratios between exciting force (or moment) and resulting pile-head displacement (or rotation), and they are complex functions of the frequency $\omega = 2\pi f$. In this paper we express each impedance as

$$\mathscr{K} = K(k + 2iD) \tag{7}$$

in which K is the static stiffness in the particular mode, $k = k(\omega)$ the dynamic stiffness coefficient, and $D = D(\omega)$ the 'effective' damping ratio of the system. D will in general consist of two components: one, which is frequencyindependent and arises from the presence of hysteretic damping in the soil, and another one, which increases with frequency and expresses the amount of radiation damping in the system.

In practice, the majority of piles subjected to lateral *head loading* are *flexible*, in the sense that they do not deform over their entire length. Instead, pile deformations and stresses reduce to negligible proportions within a distance l_a (on the order of 10 to 15 diameters) from the ground surface. We name l_a : the *active* pile length. With flexible piles, $L > l_a$, the exact pile length L is an irrelevant parameter, having no influence on the response.

To keep the number of independent problem parameters small (without seriously restricting the range of practical validity of the results) only flexible piles are studied in this section. L is kept constant, equal to 40d. Most of the presented results, however, will also be applicable with larger and even with somewhat smaller values of L, as long as $L \ge l_a$.

Table 2 presents simple expressions for estimating the active length of piles in each of the three soil profiles. These expressions were derived from the results of the finiteelement analyses and are applicable with reasonable accuracy for a fairly wide range of frequencies. At depths below $z = l_a$ a head-loaded pile would experience deformations which are within 5% of those at the top; furthermore, removing this lower (idling) part of the pile would only insignificantly affect its head impedances.

Static stiffnesses

Table 3 gives simple expressions for the (length-independent) static stiffnesses K_{HH} , K_{MM} and K_{HM} associated with each of the three studied soil Models. These expressions were obtained by fitting the static finite-element results and their excellent accuracy has been verified by comparing with the expressions of Blaney *et al.*⁴ for the case of homogeneous soil and of Randolph¹⁶ for soil with modulus proportional to depth. The simplicity of these formulae makes them particularly attractive for design computations.

 Table 2.
 Active length of pile under lateral dynamic loading at the top

	Active length la		
Soil model	Expression	Typical range	
A $(E = E_{s}z/d)$	$3.2d\left(\frac{E_p}{E_s}\right)^{1/6}$	6d — 15d	
$(E = E_s \sqrt{z/d})$	$3.2d\left(\frac{E_p}{E_s}\right)^{2/11}$	6 <i>d</i> — 17 <i>d</i>	
$C \\ (E = E_s)$	$3.3d\left(\frac{E_p}{E_s}\right)^{1/5}$	8 <i>d</i> — 20 <i>d</i>	

Table 3. Expressions for static stiffness of flexible piles embedded in three soil profiles

Soil model	$\frac{K_{HH}}{\mathrm{d}E_{sc}}$	$\frac{K_{MM}}{\mathrm{d}^{3}E_{sc}}$	$\frac{K_{HM}}{d^2 E_{sc}}$
$(E = \frac{A}{E_s z/d})$	$0.60 \left(\frac{E_p}{E_s}\right)^{0.35}$	$0.14 \left(\frac{E_p}{E_s}\right)^{0.80}$	$-0.17 \left(\frac{E_p}{E_s}\right)^{0.60}$
$(E = E_{s}\sqrt{z/d})$	$0.79 \left(\frac{E_p}{E_s}\right)^{0.28}$	$0.15 \left(\frac{E_p}{E_s}\right)^{0.77}$	$-0.24 \left(\frac{E_p}{E_s}\right)^{0.53}$
$\begin{array}{c} C\\ (E=E_{s})\end{array}$	$1.08 \left(\frac{E_p}{E_s}\right)^{0.21}$	$0.16 \left(\frac{E_p}{E_s}\right)^{0.75}$	$-0.22 \left(\frac{E_p}{E_s}\right)^{0.50}$

Dynamic stiffness coefficients and effective damping ratios

Figures 7, 8 and 9 portray the variation with frequency of the three pairs of stiffness coefficients and damping ratios, namely (k_{HH}, D_{HH}) , (k_{MM}, D_{MM}) and (k_{HM}, D_{HM}) , for each of the three soil Models, A, B and C. Several values of E_p/E_s covering a rather extreme range of practical situations and a hysteretic damping ratio in the soil equal to 0.05 are considered. The results are plotted versus f/f_1 , where f_1 = the fundamental shear frequency of the unperturbed soil deposit, given by the expressions of Table 1 for the considered profiles.

Again, the choice of f_1 as a normalizing parameter is hardly arbitrary. Even in this case of a head-loaded pile, Figs. 7-9 reveal that resonance phenomena occur almost precisely at $f = f_1$, for all soil profiles and all E_p/E_s ratios. At resonance, the dynamic stiffness coefficients experience a dip, which is especially sharp for stiff piles (i.e. with large E_p/E_s ratios). Moreover, below f_1 the effective damping ratios attain small and frequency-independent values which reflect the material (hysteretic) damping in the system. At such low frequencies, no radiation damping is present since neither surface nor body radially-propagating waves can be physically created in the soil stratum. But as soon as fexceeds f_1 , damping ratios start increasing with f due to the developing radiation damping.

It is also worth noting a few interesting trends in Figs. 7-9.

1. The variation of the dynamic stiffness coefficients with frequency is not dramatic, except perhaps for k_{HH} of relatively soft piles ($E_p/E_s \leq 1500$) in the inhomogeneous soil profiles A and B. Particularly



Figure 7. Dynamic stiffness coefficients and damping ratios for flexible piles embedded in soil Model A ($\beta_s = 0.05$; $\nu_s = 0.40$, $\rho_p/\rho_s = 1.60$)

insensitive to variations in frequency are the rocking coefficients, k_{MM} , regardless of soil profile. Assuming constant (frequency-independent) dynamic stiffnesses for design purposes will introduce only small errors in most actual situations.

- The effective damping ratios are considerably lower in the rocking than the swaying mode of vibration. This is true for all frequencies and is consistent with the well known results for shallow rigid foundations.³³
- 3. The type of variation of soil modulus with depth has a profound effect on the damping ratios, especially at high frequencies. For instance, compare the results for Models A and C. In A, as E_p/E_s increases, the rate of increase of D_{HH} with frequency drops considerably; thus, the softest piles exhibit the highest D_{HH} values at $f/f_1 \gtrsim 5$. The opposite is true with Model C.

The dimensionless graphs in Figs. 7-9 along with the formulae in Tables 1-3 make it possible to readily evaluate static/dynamic stiffnesses and damping ratios of piles in a variety of actual situations, without the need of a computer.

COMBINED KINEMATIC-INERTIAL INTERACTION

An example of a complete soil-pile-structure interaction analysis, in which the effects of both kinematic and inertial interaction are combined, is presented in Fig. 10. The example refers to the steady-state response atop a pile embedded in a linearly inhomogeneous deposit (Model A) and supporting a mass m, located just above the ground surface. This is the simplest possible model of a super-structure, which may nevertheless help highlight key aspects of the response. The natural frequency of such a super-structure is given by

$$f_{st} \approx \frac{1}{2\pi} \sqrt{\frac{m}{K_h}} \tag{8}$$

in which K_h is the static force-displacement ratio of a pile subjected to a sole horizontal force and free to rotate. K_h is related to the static stiffnesses of Table 3:

$$K_h = K_{HH} - \frac{K_{HM}^2}{K_{MM}} \tag{9}$$

The approximation in equation (8) consists in that the static instead of the dynamic force-displacement ratio is used. This is justified in view of the fact that k_{HH} , k_{HM} and k_{MM} are relatively insensitive to frequency (Figs. 7-9). Moreover, in Fig. 10, f_{st} from equation (8) is merely used as a convenient normalizing parameter; it was not used in the analyses, which are 'exact'.

Figure 10 illustrates the effect of the relative frequency factor f_{st}/f_1 , for a pile with L/d = 40 and $E_p/E_s = 29000$. A few trends are worthy of note.

The response of the soil-pile-structure system exhibits resonant peaks at two different sets of frequencies: the



Figure 8. Dynamic stiffness coefficients and damping ratios for flexible piles embedded in soil Model $B(\beta_s = 0.05, \nu_s = 0.40, \rho_p/\rho_s = 1.60)$



Figure 9. Dynamic stiffness coefficients and damping ratios for flexible piles embedded in soil Model $C(\beta_s = 0.05, \nu_s = 0.40, \rho_p/\rho_s = 1.60)$

first set corresponds to the natural frequencies of the superstructure. This is in agreement with the results of other authors.^{2,9} Moreover, the highest peaks of the displacement interaction curves, $I_u(f/f_1; f_{st}/f_1)$, always occur at $f \simeq f_{st}$, the natural frequency of the super-structure. Hence, as the mass *m* increases, the highest of the peaks tends to occur at lower frequencies of excitation where radiation damping may be particularly small (Figs. 7-9); such peaks may thus be quite sharp. On the contrary, a relatively small mass (large f_{st}/f_1) may be associated with very large damping and very flat resonant peaks.

The results for other piles and soil profiles (not shown here for lack of space) are in qualitative agreement with those in Fig. 10.

ANALYSIS OF A CASE-HISTORY

Since place and time of earthquake occurrence cannot be accurately predetermined, there is generally only a limited amount of recorded field information with which the adequacy of developed analytical methods can be judged. Moreover, the seismic response of structures founded on piles embedded in soft subsoil is a complicated problem and it is difficult to clearly delineate the role of pile-soil interaction on the response. For this reason, a well documented case history presented by Ohta *et al.*³¹ is of significance and is studied herein.

The vibration of an 11-storey apartment building supported on piles was recorded during seven earthquakes. The building consists of reinforced concrete-and-steel composite frame and rises nearly 31 m above the ground. The ground floor is of the 'pylotis' type and there is no basement. The plan and two cross-sections of the building are shown in Fig. 11.

The building is founded on cast-in-place concrete piles of about 25 m in length and 1.4 m in diameter. Two 2.3 m-high footing beams running in the longitudinal direction



Figure 10. Typical results of complete soil-pile-structure interaction analysis



Figure 11. Case history: (a) plan and section of the building; (b) location of recording devices; (c) S-wave velocity profile (after Ohta et al.³¹)

transfer the loads of the super-structure on the piles. The soil deposit is of alluvial origin and consists of alternating layers of sand and silt. Below about 25 m from the ground surface the soil becomes dense gravelly-sand and stiff clay. Prior to recording the reported ground motions several field tests had been carried out to obtain the dynamic soil properties at various depths. These tests included *SH* wave velocity measurements with the so-called Well-Shooting method and forced vibration tests on piles.³¹ Figure 11 portrays the *SH*-wave velocity profile.

Characteristics of the seven recorded seismic events are given in Table 4. These events can be roughly classified into two categories: small-magnitude, near-distant events ($M_L \leq 5$ and R < 40 km); and moderate and large-magnitude far-distant events ($M \geq 5.5$ and R > 65 km).

The response of the soil-pile-structure system during these ground shocks was monitored by means of 27 accelerometers, eight displacement-meters, four earth-pressure gauges and two porewater pressure gauges. As indicated in Fig. 11, the accelerometers were placed along three different vertical axes: on the pile-building axis; on the axis 5 m away from the pile (to be called 'nearby-soil' axis); and on an axis 35 m away from the pile (which essentially is a 'free-field' axis). A summary of the recorded results is presented below.

Figure 12a portrays the distribution of the peak absolute values of accelerations recorded along the three axes during

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the seven earthquakes. Of particular interest in this paper is the relationship between the 'free-field' ground surface acceleration, \ddot{u}_0 , and the acceleration at the head of the pile, \ddot{u}_p . Any difference between \ddot{u}_0 and \ddot{u}_p is the result of pile-soil-structure interaction, as discussed in preceding sections. Figure 12b plots the ratio of the respective peak values, max $\ddot{u}_p/\max\ddot{u}_0$, for each earthquake. It is evident that for nearby events:

$$\frac{\max \ddot{u}_p}{\max \ddot{u}_0} \approx 0.6 \tag{10a}$$

while for distant events:

$$\frac{\max \ddot{u}_p}{\max \ddot{u}_0} \approx 1 \tag{10b}$$



Figure 12. Case history: (a) distribution with z of the recorded peak accelerations; (b) variation with epicentral distance of the ratio of peak acceleration atop the pile and at the free-field ground surface

Table 4.	Case history:	recorded	earthquakes ³¹
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Earthquake No.	Year	Name	Epicentral distance (km)	Focal depth (km)	Magnitude	Maximum free-field acceleration (gal)
1	1975	Central Chiba	45	70	4.6	10.2
2	1976	Eastern Tokyo	0	40	4.2	28.6
3	1976	Eastern Saitana	40	70	4.8	22.0
4	1976	Eastern Yamonashi	65	20	5.5	27.4
5	1978	Near Izu Ohshima	110	0	7.0	21.3
6	1978	Off Mivagi	400	50	6.7	13.4
7	1978	Off Miyagi	350	40	7.4	57.4

These values are in qualitative agreement with the results of this paper. Indeed, as anticipated, the three nearby motions (not shown here) are very rich in high-frequency components,³¹ such components contribute the most to peak acceleration. Since kinematic interaction filters the high frequency components of motion (recall Fig. 6), it is natural to have peak accelerations at the pile head smaller than those at the free field. Conversely, distant records are rich in low and medium frequency components which are not as much influenced by the presence of piles; hence, the peak accelerations at the free field and the pile head are of about the same magnitude.

Furthermore, Fig. 13 plots the ratio \ddot{U}_p/\ddot{U}_0 of the Fourier Amplitude Spectra of the pile-head and the free-field acceleration records. The plot shows that, indeed, (1) high frequency components of the seismic motion are filtered out by the pile-soil-structure interaction; (2) low frequency components are not affected by the pile or structure; and (3) components in the frequency range between the fundamental frequency of the soil stratum, f_1 , and the fundamental frequency of the super-structure, f_{st} , are substantially amplified due to pile-soil-structure interaction.

Also plotted in Fig. 13 is an approximate theoretical curve derived in a simplified way on the basis of the results of this paper, as follows: first, the soil is approximated as a homogeneous 20-m thick stratum with $V_s = 110 \text{ m/s}$, $v_s = 0.40$, $\rho_s = 1.60 \text{ t/m}^3$, $E_s \simeq 54 \text{ MN/m}^2$, and a fundamental shear frequency $f_1 = 110/(4 \times 20) \approx 1.38 \text{ Hz}$. For the pile, $E_p \simeq 22\,000 \text{ MN/m}^2$, d = 1.4 m, and, effectively, L = 20 m. Therefore:

$$\frac{E_p}{E_s} \simeq 407 \quad \text{and} \quad \frac{L}{d} \simeq 14.3$$
 (11)

Since the spacing between piles is fairly large, $s \approx 8.35$ m $\approx 6d$, one may expect only small errors in the analysis due to pile-soil-pile interaction effects.^{10,11}

To access the kinematic interaction effects, the dimensionless frequency parameter F_C is obtained from equation (6):

$$F_C = \frac{f}{1.38} \times (407)^{0.30} (14.3)^{-0.50} \simeq 1.16f \qquad (12)$$



Figure 13. Case history: comparison of actually recorded and approximately predicted ratio of Fourier-Amplitude spectra atop the pile and at the free-field ground surface

The active pile length is obtained from Table 2:

$$l_a = 3.3 \times 1.4 \times (407)^{1.5} \simeq 15.4 \text{ m}$$
 (13)

which is less than L = 20 m; hence, the pile is flexible. despite its very large diameter. Table 3 may then be consulted to obtain the static stiffnesses. For example:

$$K_{HH} = 1.08 \times 1.4 \times 54 \times (407)^{0.21} \simeq 288 \text{ MN/m} (14)$$

Similarly:

$$K_{MM} \simeq 2148 \text{ MN} \cdot \text{m/rad}, K_{HM} \simeq 470 \text{ MN/rad}$$

and, from equation (9),

$$K_h \simeq 185 \text{ MN/m}.$$

A very simple model, appropriate only for preliminary design computations, is adopted for the super-structure, which is assumed to vibrate only in its first natural mode.³⁴ The corresponding first natural frequency, f_{st} , is obtained as follows: first, we estimate roughly the fixed-base small-deformation natural frequency, f_{st}^* , using an empirical formula from the literature:^{35,36}

$$f_{st}^* \approx 24 H^{-3/4} \simeq 24 (31)^{-3/4} \simeq 1.83 \text{ Hz}$$
 (15)

Because of the flexibility of the support (consisting of the pile-soil system) f_{st} will be inferior to f_{st}^* . Using Dunkerley's rule³⁴

$$\frac{1}{f_{st}^2} \approx \frac{1}{f_{st}^{*2}} + \frac{4\pi^2}{K_h/M_e} = \frac{1}{1.83^2} + \frac{4\pi^2}{185\,000/195} \simeq 0.34 \ (16)$$

from which: $f_{st} \simeq 1.71$ Hz. In equation (16) $M_e = 195$ tons is the 'effective' mass of building participating in the first mode and corresponding to each pile:³⁴

$$M_{e} = \frac{\left(\sum_{i=1}^{11} M_{i} \phi_{i}\right)^{2}}{\sum_{i=1}^{11} M_{i} \phi_{i}^{2}}$$
(17)

in which M_i and ϕ_i are the mass and modal displacement of the *i*th floor. Assuming a linear mode shape and a uniform distribution of mass along the height leads to $M_e = 0.75 \text{ M}$,³⁴ where $M \approx 260$ tons is the total mass of the building carried by each pile.

In the interest of simplicity and consistency with the overall approximation, the building is modeled as a single mass, $m = 4\pi^2 f_{st}^2 K_h$, supported on an unrestrained-head pile with $K_h = 185 \text{ MN/m}^2$, and located above the ground surface. The actual height of the super-structure, important as it is for the response of its higher storeys, has only a minor effect on the response at the base of the structure,³⁷ which is of interest here. In addition, the frequency variation of dynamic stiffnesses is also ignored.

Thus, the problem reduces to computing the response of a one-degree-of-freedom system having mass m, stiffness K_h and frequency-dependent damping ratio D_h , and being subjected to a harmonic free-field base motion (since $I_u \approx 1$). D_h is obtained at each frequency, on the basis of the graphs of Fig. 9, for $E_p/E_s = 407$. However, since the actual soil profile is not underlain by a rigid bedrock but rather by gravelly-sand and clay layers of an average velocity $V_{sr} \approx 400$ m/s, radiation damping would be underpredicted from Fig. 9. Roesset²⁹ has suggested that an additional equivalent radiation damping

$$D_r = \frac{2 V_s f_1}{\pi V_{sr} f} = \frac{2 110 1.38}{\pi 400 f} \simeq \frac{0.24}{f}$$
(18)

should be added to D_h . At $f = f_{st}$ this additional damping ratio amounts to a substantial 14%.

The resulting approximate steady-state theoretical curve for \ddot{U}_p/\ddot{U}_0 predicts very well the most important trends observed in the ratio of the recorded Fourier Amplitude Spectra. This gives confidence in the usefulness of the results presented in the paper.

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REFERENCES

- 1 Nair, K. Dynamic and earthquake forces on deep foundations, Performance of Deep Foundations ASTM STP 444, 1969, pp. 229-263
- 2 Flores-Berrones, R. Behavior of end bearing piles under seismic forces, 9th Int. Conf. Soil Mech. Fdn. Engng., Proc. Specialty Session 10, Tokyo 1977
- 3 Flores-Berrones, R. and Whitman, R. V. Seismic responses of end-bearing piles, J. Geotech. Engng., ASCE 1982, 108 (GT4), 554
- 4 Blaney, G. W., Kausel, E. and Roesset, J. M. Dynamic stiffness of piles, Proc. Second Int. Conf. Num. Meth. in Geomech., ASCE Blacksburg, Virginia, 1976, pp. 1001-1012
- 5 Takemiya, H. and Yamada, Y. Layered soil-pile-structure interaction, Int. J. Earthq. Engng. Struct. Dyn. 1981, 9, 437
- 6 Kagawa, T. and Kraft, L. M. Lateral load-deflection relationship of piles subjected to dynamic loadings, Soils and Foundations 1980, 20 (4), 19
- 7 Kagawa, T. and Kraft, L. M. Dynamic characteristics of lateral load-deflection relationships of flexible piles, *Int. J. Earthq. Engng. Struct. Dyn.* 1981, 9, 53
- 8 Harada, T., Kubo, K. and Katayama, T. Dynamic soil-structure interaction by continuum formulation method, *Report*, *Inst. Industrial Science*, *Tokyo* 1981, 29 (5), 1
- 9 Tajimi, H. Seismic effects on piles, State-of-the-Art Report No. 2, Int. Conf. Soil Mech. Found. Engng., Proc. Specialty Session 10 1977, pp. 15-26
- 10 Wolf, J. P. and Von Arx, G. A. Travelling waves in a group of piles taking pile-soil-pile interaction into account, Proc. 7th World Conf. Earthq. Engng., Istanbul, Turkey 1980, pp. 467-470
- 11 Kaynia, A. M. Dynamic stiffness and seismic response of pile groups, Research Report R82-03 Massachusetts Inst. of Tech., 1982
- 12 Nogami, T., Idriss, I. M., Power, M. and Chang, C. Y. Effect of radiation damping on earthquake response of pile supported off-shore platforms, Int. J. of Earthq. Engng. Struct. Dyn. 1983, 11 (3)
- 13 Prakash, S. Soil Dynamics, McGraw-Hill, 1981, Ch. 7

- 14 Dobry, R. and O'Rourke, M. J. Discussion on 'Seismic response of end-bearing piles' by R. Flores-Berrones and R. V. Whitman, J. Geotech. Engng., ASCE 1983, 109
- 15 Poulos, H. G. and Davis, E. H. Pile Foundation Analysis and Design, John Wiley and Sons, 1980
- 16 Randolph, M. F. Response of flexible piles to lateral loading, Geotechnique 1981, 31 (2), 247
- 17 Banerjee, P. K. and Davies, T. G. The linear behavior of axially and laterally loaded single piles embedded in nonhomogeneous soils, *Geotechnique* 1978, 28 (3), 309
- 18 Krishnan, R., Gazetas, G. and Velez, A. Static and dynamic lateral deflexion of piles in nonhomogeneous soil stratum, *Geotechnique* 1983, 33 (3)
- 19 Velez, A., Gazetas, G. and Krishnan, R. Lateral dynamic response of constrained-head piles, J. Geotech. Engng., ASCE 1983, 109 (8)
- 20 Dobry, R., Vicente, E., O'Rourke, M. J. and Roesset, J. M. Horizontal stiffness and damping of single piles, J. Geotech. Engng., ASCE 1982, 108 (GT3), 439
- 21 Novak, M. and El Sharnouby, B. Stiffness constants of single piles, J. Geotech. Engng., ASCE 1983, 109 (7), 961
- 22 Kulemeyer, R. L. Static and dynamic laterally loaded floating piles, J. Geotech. Engng., ASCE 1979, 105 (GT2), 289
- 23 Kobori, T., Minai, R. and Baba, K. Dynamic behavior of a laterally loaded pile, Proc. Specialty Session 10, 9th Int. Conf. Soil Mech. Found. Engng., Tokyo 1977
- 24 Roesset, J. M. and Angelides, D. Dynamic stiffness of piles, Numerical Methods in Offshore Piling, Inst. of Civil Engineers, London, 22-23 May 1979
- 25 Nogami, T. and Novak, M. Resistance of soil to a horizontally vibrating pile, Int. J. Earthq. Engng. Struct. Dyn. 1977, 5, 249
- 26 Whitman, R. V. and Bielak, J. Foundations, Design of Earthquake Resistant Structures, Rosenblueth, E., ed., John Wiley & Sons, 1980
- 27 Kausel, E. and Roesset, J. M. Soil-structure interaction for nuclear containment structures, Proc. ASCE Power Division Specialty Conf. Boulder, Colorado, 1974
- 28 Gazetas, G. and Dobry, R. Horizontal response of piles in layered soils, J. Geotech. Engng., ASCE accepted for publication
- 29 Roesset, J. M. Soil amplification of earthquakes, Numerical Methods in Geotech. Engng., Desai, C. S. and Christian, J. T., eds., McGraw-Hill, 1977
- 30 Dobry, R., Oweis, I. and Urzua, A. Simplified procedures for estimating the fundamental period of a soil profile, Bulletin Seismological Soc. of Am. 1976, 66 (4), 1293
- 31 Ohta, T., Uchiyama, S., Niwa, M. and Ueno, K. Earthquake response characteristics of structure with pile foundation on soft subsoil layer and its simulation analysis, Proc. 7th World Conf. on Earthq. Engng., Istanbul, Turkey 1980, pp. 467-470
- 32 Elasbee, F. and Morray, J. P. Dynamic behavior of embedded foundations, *Publication No. R77-33, MIT* 1977
- 33 Gazetas, G. Analysis of machine foundation vibrations: state of the art, Int. J. of Soil Dynamics and Earthq. Engng. 1983, 2 (1), 2
- 34 Veletsos, A. S. Dynamics of structure-foundation systems, Structural & Geotechnical Mech., Hall, W. J., ed., Prentice-Hall, 1977
- 35 Tassios, T. P. and Gazetas, G. For a Predraft of a New Greek Aseismic Code, National Technical University of Athens, Greece, 1979
- 36 Tentative provisions for the development of seismic regulations for buildings, ATC-3-06 Report, Applied Technology Council, Palo Alto, California, 1978
- 37 Richart, F. E., Jr. and Chou, C.-S. Note on stiffness and damping of pile systems, Proc. Specialty Session 10, 9th Int. Conf. Soil Mech. Found. Engng., Tokyo 1977