Stochastic estimation of the nonlinear response of earth dams to strong earthquakes

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The paper develops a simple and rational piece-wise linear random vibration procedure to estimate statistics of the nonlinear hysteretic response of earth dams to strong nonstationary stochastic excitation characterized by a Kanai-Tajimi spectrum. The method utilizes published data relating soil modulus and damping to the level of induced shear strain and incorporates a realistic inhomogeneous shear beam model of the dam. It is shown that the method is technically superior and computationally simpler than currently used iterative linear deterministic methods of analysis. Results are presented in the form of variation with time and distribution with depth from the crest of statistical measures of displacements, accelerations, shear strains and seismic coefficients on potentially sliding masses. The influence of the intensity and frequency characteristics of the excitation on the nonlinear response of a broad range of earth dams is graphically illustrated and a set of normalized graphs are presented whereby one can readily obtain preliminary estimates of key response quantities to be experienced by an earth dam during a potential earthquake motion of known rms acceleration and predominant frequency.

INTRODUCTION

Consideration of the nonlinear inelastic shearing stress-strain behavior of soil under large-amplitude cyclic loading is necessary for a realistic assessment of the safety of earth dams against strong potential earthquakes. Moreover, since most structures are quite sensitive even to the details of ground excitation*, which are of an unpredictable (random) nature, no confidence can be achieved from the results of a single deterministic dynamic analysis using a recorded or artificially generated motion. To avoid the expense and effort required for multiple deterministic analyses, probabilistic (random vibration) methods have been developed. Such methods require a stochastic description of the earthquake excitation and directly provide probabilistic information on the response, helping the designer to make rational decisions regarding the safety of the facility.

Although classical random vibration theory applies only to linear elastic systems\(^2\,^3\), extensive work and significant progress has been made in recent years towards developing random vibration methods to analyze the response of nonlinear (elastic and inelastic) systems. For such systems the response depends not only on the intensity and frequency content of the input motion, but also on the governing force-deformation characteristics and no general exact analytical solutions are presently available. A variety of approximate formulations have thus been proposed\(^4\,^1-\,^4\,^1\,^4\). The corresponding analyses offer valuable insight on the dynamic behavior of certain nonlinear/inelastic systems subjected to special types of stochastic excitation.

Few researchers have applied nonlinear random vibration techniques in computing the seismic response of continuous systems that are of interest in soil dynamics. For instance, Faccioili\(^6\) developed an equivalent linearization random vibration formulation to study the one-dimensional amplification of seismic waves by deposits of soil obeying Ramberg-Osgood-Masing constitutive equation; and Singh and Khatua\(^11\) attempted to assess the seismic safety of earth dams by using stochastic linearization and performing an iterative analysis in connection with a finite-element discretization of the dam. Recently, Vanmarcke\(^16\) suggested several possible applications of random vibration theory to solve soil dynamics problems, including determination of nonlinear soil response and assessment of liquefaction potential.

On the other hand, in spite of the progress made in recent years towards developing 3-dimensional plasticity-based constitutive laws for soils, currently used deterministic procedures utilize simple iterative linear techniques to estimate the hysteretic response of dams to strong shaking [e.g. references 17, 18, 19, 6]. Although of an approximate nature, such procedures are still attractive mainly for two reasons: they are relatively inexpensive and they can make use of a wealth of already available experimental data relating soil shear modulus and equivalent damping ratio to shear–strain amplitude. Such data have been obtained from laboratory cyclic triaxial and simple shear tests by numerous workers in the field\(^10\,^18\). Recently moreover, Abdel-Ghaffar and Scott\(^2\,^2\,^2\,^3\) analyzed records of full-scale (earthquake and man-induced) vibrations of an embankment dam in California and estimated the actual variation with level of induced strains of the soil modulus.
and damping ratio. Figure 1 summarizes the results.

It is the purpose of this paper to present a random vibration methodology to estimate statistics of the nonlinear hysteretic response of earth dams to strong stochastic excitation. The method, based on a nonstationary random vibration formulation, utilizes published soil data such as those portrayed in Figure 1 and incorporates a newly developed simple, realistic, dynamic model for the dam. It is shown that the method offers an appreciable improvement over the corresponding "strain-compatible" iterative linear deterministic methods of analysis, both from a technical and an economic viewpoint. A series of comprehensive parametric studies is, thus, quite feasible with the developed method; the results of such studies are reported in the paper. Key factors that exert a sizeable influence on the dynamic response of a dam are identified and their effects graphically demonstrated. On the basis of these results a set of normalized graphs are constructed whereby one can readily obtain estimates of crest accelerations and shear strains to be experienced by a dam of known geometry and (average) low-amplitude shear modulus during strong earthquake shaking of known intensity and predominant frequency. These graphs may serve as a guide in preliminary design calculations.

BACKGROUND: STRAIN-COMPATIBLE ITERATIVE LINEAR VISCOELASTIC ANALYSIS

Because of the lack of a constitutive soil model for general use in soil dynamics and, moreover, the prohibitive expense of a step-by-step numerical integration of the nonlinear equations of motion, truly nonlinear-inelastic deterministic analyses are considered impractical in all but exceptionally critical cases. A practical solution to the problem is offered by a method known as the "equivalent linear" method. Despite its empirical nature it is widely used at present and has been implemented in such popular computer codes for seismic response analyses of soil systems as SHAKE, and QUAD-4.

According to this method an approximate solution can be obtained by a linear viscoelastic analysis provided the soil stiffness and damping used are compatible with the level of induced shearing strains. This method utilizes the results of laboratory tests such as those of Figure 1 where shear modulus and damping ratio are displayed as functions of the imposed sinusoidal shear-strain amplitude. An iterative computational procedure is then implemented as follows: A set of moduli and damping values is initially estimated for each soil element and the system is analyzed linearly using these properties. The solution provides the time-history of the shear strain at each element from which an "equivalent" or "effective" strain amplitude \( \gamma_e \) is estimated and Figure 1 is consulted to check if the strain level is compatible with the values of shear modulus and damping used in the analysis. If they are not compatible, a new analysis is performed using the soil properties corresponding to \( \gamma_e \) and the process is repeated until convergence has occurred (usually within five iterations). The response of the last iteration is taken as being the nonlinear response.

A crucial step in the analysis is to deduce the amplitude \( \gamma_e \) of a sinusoidal strain history "equivalent" to the erratic time-history of the computed strain. To this end, a strain reduction factor (SRF), often taken to be on the order of 2/3, is applied to the peak shear strain amplitude. Clearly the arbitrary selection of an SRF is, along with its linear character, a major drawback of the method. Peak strain amplitudes, important as they may be, often bear very little relation to the overall level of deformation; they are just spikes occurring instantaneously while during the rest of the time deformation amplitudes may or may not be of a comparable magnitude. Therefore it is quite possible for this method to lead to an artificially overdampered system and thus underestimate the response, or, vice versa, in case of a relatively uniform motion, to yield very conservative response estimates.

The nonlinear random vibration formulation that is presented in this paper largely avoids these drawbacks of the deterministic iterative linear method.

STRAIN-COMPATIBLE PIECE-WISE LINEAR STOCHASTIC ANALYSIS OF EARTH DAMS

Dynamic model of dam and stochastic excitation

Although the random vibration methodology can accept any sophisticated dynamic model of the earth structure, the relatively simple "inhomogeneous shear beam" model, recently developed by one of the authors, has been preferred to the somewhat more accurate "plane-strain finite-element" model. The choice was based not merely on economic considerations but, also, on the very good performance of the shear model in interpreting the observed behavior of actual dams during earthquakes and forced vibration experiments.

The model treats the dam as a triangular or truncated wedge-shaped shear beam and assumes that: (1) only horizontal shear deformations take place, and (2) the resulting horizontal displacements and shear stresses are uniformly distributed on horizontal planes. Furthermore,

* More accurately: "strain-compatible iterative linear viscoelastic" method.
* In agreement with classical (homogeneous) shear beam theory.
the model accounts for the dependence of soil stiffness on effective confining pressure by considering that the average shear modulus \( G \) across a horizontal plane increases as the \( 2/3 \)-power of the depth from the crest. Such a variation of \( G \) has been substantiated by theoretical analyses\(^{27}\) and ample field evidence\(^{27,29}\). Only the expressions for natural periods \( T_n \) and natural modal displacements \( \Phi_n \) of a tall dam (i.e. with negligible truncation) are reproduced here:

\[
T_n = \frac{18H}{7nC}; \quad n = 1, 2, \ldots
\]

\[
\Phi_n = \frac{\zeta - 2^{1/3}}{\sin \left[ \pi n \left( 1 - \zeta^{2/3} \right) \right]; \quad \zeta = \frac{z}{H}; \quad n = 1, 2, \ldots
\]

in which \( H \) = height of dam, \( z \) = depth from crest (Figure 2a) and \( C \) = average S-wave velocity of the dam.

The ground oscillation, triggered exclusively by vertically propagating S-waves, is stochastically described through a time-independent Kanai-Tajimi power spectral density function (Figure 2b) and an intensity function varying with time (Figure 2c). This is a fairly realistic description of earthquake excitation, since it models the nonstationarity and the nonuniform distribution of power observed in real earthquakes. To define the excitation one must specify the "predominant ground frequency" \( \omega_f \), the "ground damping" \( \zeta_f \), the zero-frequency ordinate of the power spectral density (psd) \( S_0 \), and the shape of the intensity function. If pertinent statistical data are not available for the particular site, \( \omega_f \) can be approximated with the natural frequency of the underlying soil deposit or taken equal to 6\( \pi \) in case of a dam built on rock\(^{32}\). \( \zeta_f \) has a secondary influence on the response\(^1\) and a (single) value of 0.40 seems to be appropriate in most cases\(^{32}\). The shape of the intensity function shown in Figure 2c appears to be fairly representative of many recorded motions and has therefore been used in this study. The reader is referred to previous publications by the authors\(^26,1\) for a detailed evaluation of the effect of various intensity functions on the elastic response of earth dams.

**Proposed method of analysis**

The basis of the analysis is the linear elastic random vibration formulation described in references 24–26 and 1.

Being analogous to conventional (deterministic) modal time-history analysis, this time-domain stochastic formulation yields directly the evolution with time of the root-mean-squared (rms) values of all pertinent (elastic) response quantities, such as accelerations, displacements, shear stresses and strains at every point in a dam, as well as seismic coefficients on potentially sliding masses from a dam's slopes.

Of particular interest here is the distribution of shear strains \( \gamma \) along the depth from the crest, during shaking. On the basis of many parametric studies (both probabilistic and deterministic) reported in references 1 and 28, respectively, it has been concluded that the actual distribution of shear strains may at any time be approximated, with reasonable accuracy, by a uniform distribution, which implies a constant shear strain amplitude throughout the dam. Figure 3 displays a typical such variation of \( \gamma (t) \) with depth and the suggested straight line approximation.

During strong ground shaking the dam's constituent soils develop nonlinear shearing stress-strain relationships that are described in terms of the variation of secant modulus and damping ratio with amplitude of imposed (sinusoidal) shear strain \( \gamma_s \) as in Figure 1. Consequently, the preceding observation of nearly constant \( \gamma \) throughout the dam at any time, implies that soil properties would change almost uniformly with depth whenever the strain exceeds \( 10^{-4} \gamma_s \) according to Figure 1. Hence, the increase of modulus in proportion with the \( 2/3 \)-power of the depth and the existence of a single damping ratio for the whole dam, which constitute the basis of the previously outlined dynamic model, remain approximately valid even when large nonlinear deformations take place. Thereby, the essentially one-dimensional "inhomogeneous shear beam model" can be used with a reasonable accuracy to perform strain-compatible piece-wise linear analyses. The details of such an operation are as follows:

**Figure 3. Typical distribution of rms value of shear strain with depth from crest and straight line approximation**
The duration of shaking is divided into $n$ intervals $(t_i, t_{i+1}), i = 0, 1, ..., n$. Linear random vibration analyses are performed in each time interval with soil properties $G_i$ and $\zeta_i$, compatible with the level of strain at time $t_i$, i.e. at the beginning of the interval. In other words, an updating of soil properties is enforced at $n-1$ discrete points on the time axis, in contrast with the single after-the-analysis updating of the aforementioned deterministic strain-compatible iterative method. Moreover, the equivalent sinusoidal shear strain amplitude $\gamma_{eq}$ at time $t_i$ can now be rationally evaluated by equating the temporal strain energies of a sinusoidal and the random motion. Since a sinusoidal motion of amplitude $\gamma_{eq}$ carries an energy proportional to $\gamma_{eq}^2/2$, while the corresponding value for a zero-mean random strain history is proportional to the mean squared value $\langle \gamma^2(t) \rangle = \langle [\gamma(t)]^2 \rangle$, where $\gamma$ denotes the rms value of $\gamma$, the energy equation takes the form:

$$\frac{\gamma_{eq}^2}{2} = \langle [\gamma(t)]^2 \rangle, \quad i = 1, 2, ..., n-1$$

This equation yields the “equivalent” shear-strain amplitude $\gamma_{eq}$ at time $t_i$, once the evolution of the rms value of $\gamma$ is known in the time interval $(t_{i-1}, t_i)$. The value of $\gamma_{eq}$ is then used to read $G_i$ and $\zeta_i$ from the appropriate curves of Figure 1, and a new (elastic) random vibration analysis starts for the time interval $(t_i, t_{i+1})$ with these updated soil properties.

It is noted that in establishing the “equivalent” strain it is not necessary to resort to arbitrary scaling of the computed strain values as is the case with the deterministic iterative method which applied a strain reduction factor on the computed peak value of strain. Furthermore, no deterministic time-history analysis scheme corresponds to the proposed piece-wise linear stochastic method is possible; knowledge of the exact value of the shear strain $\gamma(t)$ is of little use, since there is equal likelihood that $t_i$ corresponds to a local peak or to zero-crossing of the strain history. Thus, determination of an “equivalent” sinusoidal strain on the basis of $\gamma(t)$ is impossible.

**Convergence and accuracy of solution**

With an intensity function consisting of smooth curves as in Figure 2c, it is expected that the proposed method may in general somewhat over-estimate the response during the initial building-up period of the motion, because the updating of soil properties is based on the lower (not the average) value of $\gamma$ in each time interval of this period. This would certainly lead to an under-estimation of the “actual” effective damping in the system and, thus, most probably, to an over-estimation of the response. Conversely, the response may in general be under-estimated during the decaying period of the motion, in which the updating is based on the higher value of $\gamma$ in each time interval. It is believed that such inaccuracies at the initial and final portion of the excitation are insignificant, from a practical viewpoint, given the uncertainty regarding the exact shape of the intensity function. Moreover, such problems may be appreciably overcome, by reducing the time interval of computation. Quick convergence to the “correct” response is expected in the strong-intensity period of the motion, during which the dam would most probably attain its peak response.

To investigate the convergence of the proposed procedure as the time interval $\Delta t = t_{i+1} - t_i$ approaches zero, a series of parametric studies were conducted. Five earth dams, all characterized by a low-amplitude average shear wave velocity $V_s = 400$ m/sec but with different heights ranging from 30 m to 180 m were subjected to a strong stochastic excitation† characterized by $\omega = 6\pi$, $\zeta = 0.04$, $S_0 = 200$ cm$^2$/sec$^3$ and the intensity function depicted in Figure 2c. The shear modulus and damping ratio were assumed to vary with shearing strain amplitude in the form suggested by Makiisi and Seed and plotted in Figure 1. Four time steps were considered: $\Delta t = 1$, 0.5, 0.25, and 0.1 seconds.

A typical set of results are shown in Figure 4, where the variation with time of the rms values of crest acceleration, crest displacement, seismic coefficient at midheight and (spatially) average shear strain of the 90 m-tall dam are plotted for the aforementioned four values of $\Delta t$. The rapid convergence of all response quantities is evident, as only minor discrepancies exist among the curves corresponding to $\Delta t \leq 0.5$. In fact, even with such a large time interval as $\Delta t = 1$ sec one can estimate reasonably well the response during the strong motion period, i.e. during the horizontal portion of intensity function. Notice, also, that accelerations exhibit greater sensitivity to $\Delta t$ than strains and displacements do. Nevertheless, a step-size $\Delta t = 0.5$ sec appears to be adequate for all response quantities (expected error less than 5\%) The reader should contrast this value of $\Delta t$ with the 0.02 or 0.01-seconds step size required in deterministic time-integration analyses. The economic advantage of the proposed methodology is evident.

**RESULTS OF PARAMETRIC STUDIES**

The method discussed has been used to perform a series of analyses in order to study the evolution with time and the distribution in space (within the dam) of rms values of accelerations, displacements, strains and seismic coefficients experienced by several dams subjected to strong stochastic excitation. The results, summarized in Figures 5–8, demonstrate the importance of nonlinear soil behavior and illustrate the effects of the intensity and

* This may not always be the case as, at the same time, the effective modulus of the system is overestimated by this procedure; resonance phenomena may thus be artificially introduced and overcompensate for the increased damping.

† It has been shown that motions similar to the El Centro 1940 NS record (peak acceleration 0.33 g) can be realizations of the considered stochastic excitation process.
Figure 5. Nonlinear versus linear rms response of a 90-m-tall earth dam (numbers in parenthesis denote S-wave velocity in m/sec and damping ratio used in linear analyses)

Figure 6. Effect of excitation intensity parameter $S_0$ on nonlinear response of a 90-m-tall earth dam ($S_0$ values are in cm/sec)

Figure 7. Effect of predominant ground frequency on nonlinear response of a 90-m-tall earth dam

Figure 8. Effect of height of dam on nonlinear response ($H$ in meters)

Soil Dynamics and Earthquake Engineering, 1982, Vol. 1, No. 1
frequency characteristics of the excitation as well as of the geometry and material properties of the dam on the response. In all cases considered it is assumed that the dependence of modulus and damping on strain is described by the curves used\(^\text{17}\) (Figure 1), the intensity function is the one shown in Figure 2c and the ground damping is \(\xi_g = 0.40\) (“average” value\(^\text{32}\)). Studies on the effect of different intensity functions and ground damping values have already been presented (in connection with linear viscoelastic analyses)\(^\text{26,1}\).

**Nonlinear versus linear analyses**

Very often a single linear viscoelastic analysis is performed to estimate the nonlinear hysteretic response of an earth structure to a strong earthquake. Key to the success of such an analysis is the appropriate selection (based on available experience) of moduli \(G\) and damping ratios \(\xi\) that are consistent with the overall intensity of shaking. Figure 5 demonstrates how difficult it is to predict all nonlinear response quantities with a linear analysis.

Results of a nonlinear and several linear analyses are portrayed in Figure 5 in the form of evolution with time and distribution with depth of the rms values of displacement, \(d\), acceleration, \(\ddot{d}\), shear strain, \(\gamma\), and seismic coefficient, \(k\), experienced by a 90-m-tall earth dam with low amplitude s-wave velocity of 400 m/sec due to an \(S_0 = 200\) cm\(^2\)/sec and \(\omega_f = 6\pi\) excitation.

Since for such a strong excitation experience\(^\text{21,22,1}\) suggested that an effective shear strain amplitude of about 0.05% may develop, by consulting the appropriate modulus-damping-strain curves of Figure 1 the values \(C_s = 275\) m/sec and \(\xi_s = 12\%\) were initially chosen. As is evident from Figure 5, the corresponding elastic analysis over-predicted the displacements and strains (by about 60\%), while it under-predicted the accelerations and seismic coefficients throughout the dam. Two more linear analyses were performed using \(C_s = 200\) m/sec, \(\xi_s = 15\%\) and \(C_s = 350\) m/sec, \(\xi_s = 9\%\), respectively.

The results, also compared in Figure 5 with those of the nonlinear analysis, show the difficulty in closely predicting all response quantities with a single linear analysis. Nonetheless, the results of the last analysis (350 m/sec, 9\%) are in fair agreement with the nonlinear response.

It should be made clear, however, that the preceding comparison between nonlinear and linear analyses pertains only to rms values and does not extend to other significant probabilistic response measures, such as the “rate of zero crossings”, which strongly influence the performance of the structure\(^\text{16,3}\).

**Effect of ground intensity**

The zero-frequency ordinate, \(S_0\), of the psd function is a convenient measure of ground motion intensity; its effect on the response of a 90 m-high, 400 m/sec-stiff dam is parametrically investigated in Figure 6. As the “ground frequency" \(\omega_f\) is kept constant, equal to \(6\pi\), by varying \(S_0\) from 100 cm\(^2\)/sec\(^3\) up to 400 cm\(^2\)/sec\(^3\), the standard deviation (rms value) of ground acceleration increases from

\[
\ddot{a}_g = \frac{\pi S_0 \omega_f}{4\xi_f^2} \left(1 + 4\xi_f^2\right)^{1/2}
\]

or

\[
\ddot{a}_g = 0.1 \cdot 100 \cdot 6\pi \times 0.40 \cdot (1 + 4 \cdot 0.40^2)^{1/2} = 0.08g
\]

up to 0.16g (\(g = \text{acceleration of gravity}\)). Recalling that peak accelerations are typically (2.5 to 4) \(a_g\), one realizes that we are dealing with moderately to very strong excitation.

The effect of the increasing importance of soil nonlinearities as \(S_0\) increases should be noticed in Figure 6. For instance, the crest-to-ground amplification ratio for the rms accelerations, \(AR = \ddot{a}_c/\ddot{a}_g\) decreases from about 5 during the \(S_0 = 100\) cm\(^2\)/sec\(^3\) excitation to about 3.7 during the \(S_0 = 400\) cm\(^2\)/sec\(^3\) one. At least two factors contribute to such differences: increased effective damping and reduced stiffness of the dam as intensity of shaking increases.

**Effect of “ground frequency” \(\omega_f\)**

Figure 7 shows the influence of \(\omega_f\) on the response of a 90 m high earth dam having \(C = 400\) m/sec and subjected to an \(S_0 = 200\) cm\(^2\)/sec\(^3\) excitation. Five curves are shown on each plot, corresponding to \(\omega_f\) values of \(\pi, 2\pi, 4\pi, 6\pi\) and \(\infty\). (The latter implies a white noise psd function.) The significance of \(\omega_f\) is evident. Not only does the dam experience different crest displacements and accelerations and different strain levels during motions with different \(\omega_f\), but, moreover, the distribution of these quantities within the dam appears to be quite sensitive to \(\omega_f\).

Especially interesting is the dependence of the vertical distribution of the seismic coefficient on \(\omega_f\); the designer may not even suspect the possibility of shallow, near-crest sliding failures or intolerable permanent deformations if he, incorrectly, bases his analyses on earthquake motions having predominant frequencies less than or about equal to the fundamental frequency of the dam, since he thus under-estimates the amount of energy received by higher modes. As is well known such modes contribute significantly to accelerations near the crest.

**Effect of dam’s height**

Finally, Figure 8 compares the response of five dams with heights ranging from 15 m to 180 m. All dams have a low-amplitude velocity \(C = 400\) m/sec and are subjected to the same stochastic excitation: \(\omega_f = 6\pi\), \(S_0 = 200\) cm\(^2\)/sec\(^3\). Some noteworthy observations are as follows:

Firstly, for relatively tall dams (e.g., \(H \geq 60\) m), increasing the height of the dam tends to increase its displacement (relative to the ground) and decrease it (absolute) acceleration — a result already known to apply to all types of dynamic systems. Secondly, although very short dams (e.g., \(H = 15\) m) experience milder accelerations than some taller ones do, they develop quite large seismic coefficients on deep (i.e. with \(z \geq 0.3\) \(H\)) potential slide masses. This is apparently due to the insignificant participation of higher modes in the response of such stiff structures. Thus, accelerations developing at various points in the dam are in phase (first-mode accelerations) and they “sum up” to large seismic coefficients. On the contrary, higher modes contribute to large but out-of-phase accelerations along the height of tall, flexible dams, thus resulting in relatively small seismic coefficients.

Finally, it is clear from Figures 6–8 that the original assumption of uniform distribution with depth of rms shear strains is a very realistic simplification.

**SYNTHESIS OF RESULTS AND CONCLUSION**

On the basis of the presented (as well as some additional) parametric studies Figures 9 and 10 have been
constructed to portray, in a normalized form, the dependence of the rms values of crest accelerations and (spatially) average shear strains upon the dimensionless frequency ratio \( \omega_1/\omega_c \); \( \omega_1 \) denotes the small-amplitude fundamental natural frequency of the dam. Both figures are based on the variation of soil modulus and damping with strain suggested in reference 17.

The normalized variables of the two graphs are, respectively:

\[
A = \frac{\tilde{\alpha}_c}{\tilde{\alpha}_c} \quad \text{(units: (m/sec^2)^1/4)}
\]

\[ (5) \]

and

\[
S = \frac{\gamma_g}{\tilde{\alpha}_c H} \quad \text{(units: m}^{-1})
\]

\[ (6) \]

Equations (5) and (6) were arrived at by trial and error so that the results would fall within a narrow band. Indeed, the data points in each figure define an “average” curve with reasonably small scatter. The two graphs can be readily used to obtain preliminary estimates of the maximum (in time) rms value of crest acceleration \( \tilde{\alpha}_c \) and (spatially) average shear strain \( \tilde{\gamma}_g \), to be experienced by a dam of known height and stiffness due to a strong earthquake excitation with an rms acceleration \( \tilde{\alpha}_p \); predominant frequency \( \omega_p \) and variation of intensity with time similar to the one in Figure 2c. Thus, the two figures can serve as useful guides in preliminary design calculations.

In conclusion, the developed “strain-compatible piecewise linear” random vibration formulation offers a simple, approximate yet quite rational method to obtain statistics of the nonlinear hysteretic response of earth dams to strong earthquakes. Its technical and economic (in terms of computer storage and time) superiority over the currently used “strain-compatible iterative linear” deterministic methods of analysis has been vividly demonstrated. The method is more than an order of magnitude cheaper compared to currently used procedures. Thus, extensive parametric studies like the ones presented in the paper are quite feasible.

Nevertheless, it must be remembered that only a portion of the total response uncertainty is treated with this method. Variability in the response arising from either the uncertain knowledge and variability of the dynamic soil properties or the uncertainty in intensity and frequency content of the excitation, are obviously not directly accounted for with this method. Research is currently underway towards a formulation that will explicitly include all significant uncertainties.

ACKNOWLEDGMENT

The financial support of the U.S. National Science Foundation (Grant PFR 80-17684) is kindly acknowledged.

NOMENCLATURE

\( a \) acceleration
\( d \) displacement
\( k(\zeta) \) seismic coefficient on a triangular potential slide mass extending from the top to a depth \( z = \zeta H \) (Figure 2a)
\( \gamma \) shear strain
\( \gamma_c \) equivalent sinusoidal shear strain amplitude
\( C \) average shear wave velocity of earth dam
\( C_0 \) low-amplitude (\( \gamma \approx 10^{-6} \)) value of \( C \)
\( G \) shear modulus of soil
\( H \) height of dam (Figure 2a)
\( T_n \) nth natural period of dam (equation (1))
\( \omega_1 \) fundamental natural frequency of dam
\( \omega_2 \) low-amplitude (\( \gamma \approx 10^{-6} \)) value of \( \omega_1 \)
\( \Phi_n \) nth natural modal displacement shape of dam (equation (2))
\( \omega_p \) “predominant ground frequency”
\( \zeta \) damping ratio
\( S(\omega) \) power spectral density function of frequency
\( S_0 \) zero-frequency value of \( S(\omega) \)
\( z \) depth from the crest
\( \zeta \) \( z/H \)

The above response quantity denotes its rms value; the subscripts \( c \) and \( g \) refer to crest and ground shaking parameters, respectively.

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46 Soil Dynamics and Earthquake Engineering, 1982, Vol. 1, No. 1